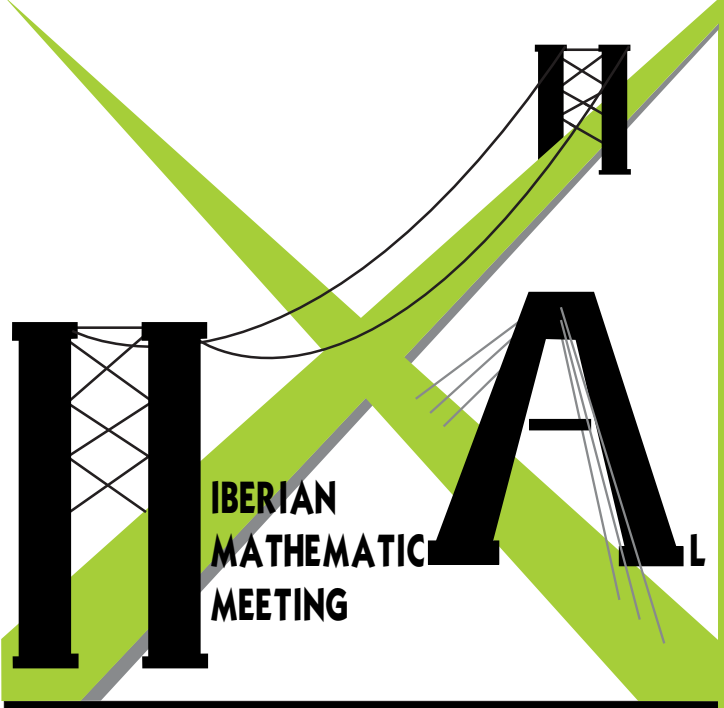

BOOK OF ABSTRACTS

II IBERIAN MATHEMATICAL MEETING

Badajoz, October 3-5, 2008.

II Iberian Mathematical Meeting

Badajoz, 3-5 october, 2008

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The second Iberian Mathematical Meeting is jointly organized by the Portuguese Mathematical Society (SPM) the Spanish Royal Mathematical Society (RSME) and the Department of Mathematics of the University of Extremadura.

Presentation

In January 2007, the Department of Mathematics of the University of Extremadura accepted the kind invitation from the President of the Spanish Royal Mathematical Society to organize the Second Iberian Mathematical Meeting in our Faculty of Science. In the hope of serving the Iberian mathematical community usefully, and delighted to receive friends and colleagues from a wide range of universities and research institutions within Portugal and Spain, an organizing committee was formed by a group of colleagues within the Department of Mathematics to whom I am most grateful for their enormous enthusiasm and selfless dedication in the organization of the meeting. Since then, taking over the baton from the Organizing Committee of the First Iberian Mathematical Meeting held in Lisbon in February of 2007, the Organizing Committee has worked relentlessly to ensure that this scientific meeting provides a venue in which to study in depth, and resolve, mathematical problems, promote academic and personal contacts between Portuguese and Spanish researchers and research teams, and contribute towards the consolidation of the Iberian Mathematical Meetings. With the aim of meeting these objectives, and in close collaboration with colleagues in the Organizing Committee, an intensive program of activities has been put together, which we hope will be found to be interesting and attractive, consisting of six plenary conferences, twelve parallel sessions and, as a new feature, about forty posters, addressing the three selected themes of the meeting; namely algebra and algebraic methods, functional analysis, and statistics and biometry. As complementary activities, two round table discussions have also been organized: the first, to be held on Friday morning prior to the official inauguration of the meeting, considering the OECD's Programme for International Student Assessment (PISA), and the second addressing the present situation of, and future perspectives for, Iberian mathematical research journals.

In the name of the Organizing Committee, it is a great honour for me to welcome you to the University of Extremadura, a young yet firmly consolidated, dynamic and diverse institution, and to the city of Badajoz, twinned with the Portuguese towns of Elvas, Santarém and Nazaré, and an important strategic enclave, due to its geographical location, for relations between Portugal and Spain, a town in which I am sure you feel most at home.

Finally, I would like to finish these few words of presentation by thanking all those collaborating institutions and organizations which, through their financial or logistic support, have made possible the organization of this the Second Iberian Mathematical Meeting.

Manuel Molina Fernández
President of the Organizing Committee

FRIDAY 3:

11.00-12.30 Round table
12.30-13.00 Opening ceremony

16.00-16.30 Registration
16.30-17.30 Plenary talk 1: Prof. José Gómez-Torrecillas
17.30-18.00 Coffee break
18.00-19.30 Parallel session 1:

Algebra and Algebraic Methods	**Functional Analysis**	**Statistics and Biometry**
Prof. Joana Ventura	Prof. Antonio F. dos Santos	Prof. Carlos Tenreiro
Prof. Carlos A.M. André	Prof. José Bonet	Prof. Arthur Pewsey
Prof. Francesc Perera	Prof. Carlos Correia Ramos	Prof. Rafael Bravo de la Parra

Poster session 1 (from 16:00 to 19:30)

20.00 Reception

SATURDAY 4:

9.00-10.00 Plenary talk 2: Prof. Jorge Mateu
10.00-11.30 Parallel session 2:

Algebra and Algebraic Methods	Functional Analysis	Statistics and Biometry
Prof. Amelia Álvarez-Sánchez	Prof. Manuel Maestre	Prof. María Ivette Gomes
Prof. Antonio Rojas-León	Prof. José Petronilho	Prof. Gerardo Sanz
Prof. Mario Jorge Edmundo	Prof. Ignacio Villanueva	Prof. Luís Meira Machado

11.30-12.00 Coffee break
12.00-13.30 Parallel session 3:

Algebra and Algebraic Methods	Functional Analysis	Statistics and Biometry
Prof. Pedro V. Silva	Prof. Jesús Bastero	Prof. Ana F. Militino
Prof. Antonio Malheiro	Prof. Antonio M. Caetano	Prof. Susana Vinga
Prof. Pedro A. García-Sánchez	Prof. Fernando Cobos	Prof. Carmen Cadarso-Suárez

Poster session 2 (from 10.00 to 13.30)

16.00-17.00 Plenary talk 3: Prof. M. Jesús Carro
17.00-17.30 Coffee break
17.30-18.30 Plenary talk 4: Prof. Eduardo Marques de Sá
18.30-20.00 Round table

Poster session 3 (from 16:00 to 19:30)

21.00 Social Dinner

SUNDAY 5:

9.00-10.00 Plenary talk 5: Prof. Antónia Amaral Turkman
10.00-11.30 Parallel session 4:

Algebra and Algebraic Methods	**Functional Analysis**	**Statistics and Biometry**
Prof. Josep Álvarez-Montaner	Prof. Jorge J, Betancor	Prof. Carlos A. Braumann
Prof. Fernando Montaner	Prof. Frank Olme Speck	Prof. Carlos Daniel Paulino
Prof. Christian Lomp	Prof. Alexei Karlovich	Prof. Jesús López-Fidalgo

11.30-12.00 Coffee break
12.00-13.00 Plenary talk 6: Prof. Stefan Samko
13.00-13.30 Closing ceremony

Poster session 4 (from 10.00 to 13.30)

Plenary talks

II Iberian Mathematical Meeting
Badajoz, 3-5 October, 2008

Salón de Grados
SATURDAY 4, 16.00

New results and applications of the extrapolation theory

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Abstract

In 1951, Yano proved that if T is a sublinear operator such that

$$T : L^p(\mu) \to L^p(\mu)$$

is bounded with constant $1/(p-1)^m$ with μ a finite measure, then

$$T : L(logL)^m(\mu) \to L^1(\mu)$$

is bounded.

Since then, a theory called "Extrapolation theory" has been developed and many interesting results have been proved. In particular, it is very useful to obtain end-point estimates.

The purpose of this talk is to give a sight of the latest results in this theory together with some new applications of it.

Salón de Grados
FRIDAY 3, 16:30

Corings. Examples and Structure

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Abstract

This talk will consist of an overview on recent developments that reveal striking connections between the theory of corings and some aspects of the algebraic approach to Non Commutative Geometry, including Descent Theory, Galois Extensions, Differential Calculi or (quantum) Principal Bundles. Some results on the structure of the category of comodules over a coring will be discussed.

Salón de Grados
SATURDAY 4, 17:30

Problems and results in algebra and combinatorics

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Abstract

Abstract. We shall considerer some problems in the theories of invariant factors of matrices over principal ideal domains, and Kronecker invariants of matrix pencils. A short reference is made to a convexity result and its extension to modular lattices. A special reference to the combinatorial flavor of these problems will be given.

Salón de Grados
SATURDAY 4, 9:00

Spatial and spatio-temporal dependencies: an excursus through biometrical applications

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Abstract

In recent years, it has become more prevalent the study of some phenomena across both space and time. Consequently, space-time data are being collected in many studies in an increasing number of disciplines. The approach taken in this context is to broaden the classical models and methods to those that recognize the presence and importance of spatial and spatio-temporal information.

Spatial statistics is a vast subject, in large part because spatial data are of so many different types: (a) univariate or multivariate, (b) categorical or continuous, (c) real-value (numerical) or not real-valued, (d) observational or experimental. In addition, the data locations may: (a) be points, regions, line segments, or curves, (b) be regularly or irregularly spaced, (c) regularly or irregularly shaped, (d) belong to a Euclidean or non-Euclidean space. We can distinguish up to three important prototypes: Geostatistical data, Lattice data, and Spatial point patterns. However, the distinctions between these three types are not always clearcut. In the geostatistical case, data corresponds to point observations of a continuously varying quantity over a region, whereas in the point pattern case, the important variable to be analysed is the location of events. The question of interest is whether the pattern is exhibiting complete spatial randomness, clustering, or regularity.

The analysis of the spatial structure is carried out through the analysis of the *large-scale structure* (mean function of a geostatistical process or intensity of a spatial point process), and the analysis of the *small-scale structure* (variogram, covariance function of a geostatistical process, and nearest-neighbour functions for spatial point processes).

The talk will consider essentially the analysis of point patterns and geostatistical data with particular emphasis on solving real problems. The **point pattern** part will consider the following points:

P1 We envisage some parametric procedures for the analysis of replicated spatial point patterns ([4]).

P2 We analyse the space-time interdependency of point processes by developing multi-generation point processes to model complex ecological systems ([2]).

P3 We propose the use of the inhomogeneous pair correlation function in the context of replicated spatial data ([3]). In addition, we analyse forest thinning strategies through the development of space-time growth-interaction simulation models ([9]).

P4 We consider the problem of detecting features of general shape in spatial point processes in the presence of substantial clutter. We use a method based on local indicators of spatial

association (LISA) functions, particularly on the development of a local version of the product density which is a second-order characteristic of spatial point processes ([5]).

P5 We develop new tools to recognise and mimic particular biological objects ([10]).

The **geostatistical** part will analyse the following aspects:

G1 We propose a new mixture-based modelling for space-time data ([7]).

G2 We build new stationary and nonstationary nonseparable space-time covariance models ([8]).

G3 We propose new models for anisotropic space-time data ([6])

G4 We propose a new procedure for the estimation of space and space-time covariance functions through a weighted composite likelihood approach ([1]).

G5 We introduce the problem of dealing with functional data within the geostatistical context.

Keywords: Forest thinning, Inhomogeneity, LISA functions, Replicated spatial data, Simulated annealing, Space-time covariance functions, Space-time separability, Weighted composite likelihood.

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Salón de Grados
SUNDAY 5, 12:00

Weighted Estimations in Variable Exponent Analysis on Metric Measure Spaces

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Abstract

Last decade there was observed an increasing interest to function spaces whose main characteristics may vary from point to point, such as the Lebesgue spaces $L^{p(x)}$, Sobolev spaces $W^{m,p(x)}$. Morrey spaces $L^{\lambda(x),p(x)}$ or Holder spaces $H^{\lambda(x)}$ and others. They may be considered on domains in the Euclidean spaces, on surfaces, on homogeneous groups and on metric measure spaces in general. In this talk there are discussed problems of weighted boundedness of the classical operators of harmonic analysis (maximal, singular and potential operators) in such variable spaces on metric measure spaces, both in homogeneous and non-homogeneous cases

Salón de Grados
SUNDAY 5, 9:00

Predictive tools in the assessment of diagnostic tests*

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Abstract

In clinical research, diagnostic tests are often used to assess disease status or other conditions. More than often, both the response variable, which is used to define the status of interest, and the variable (or variables) which are used as diagnostic tests, are continuous in nature. In most situations, comparison among different tests is done using ROC analysis based on sensitivity and specificity of the tests. However, while these measures are a good tool for discriminatory purposes, they are not in general good tools for predictive purposes. In this case, the usefulness of the diagnostic tests in competition should be based in the predictive values. The methodology behind the construction of optimal screening methods ([4]) is a basic tool to reach this goal in a variety of different situations. With those ideas in mind, a Bayesian method designated as "ROCP", ([2]) is suggested to compare the performance of tests based on their predictive capability, which generalizes the ideas behind the construction of the ROC curves. This methodology is compared with the "Skill Plot" introduced by Briggs and Zaretzki ([1]) in an attempt to answer the question "Are there analogs of ROC curves that generalize the notions of predictive values to continuous tests?" An application is done for diagnostic tests for respiratory insufficiency in amyotrophic lateral sclerosis ([3]).

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Parallel Sessions

II Iberian Mathematical Meeting
Badajoz, 3-5 October, 2008

Algebra and Algebraic Methods. Aula 09
SUNDAY 5, 10:00

D-modules in positive characteristic and Frobenius descent

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Abstract

Let $R = k[x_1, \ldots, x_d]$ be the ring of polynomials in a finite number of variables over a field k and let $D_{R|k}$ be the corresponding ring of k-linear differential operators. The theory of $D_{R|k}$-modules has been successfully applied in Commutative Algebra in order to study local cohomology modules due to the fact that, despite not being finitely generated as R-modules, they are so when considered as modules over $D_{R|k}$.

When k is a field of characteristic zero, $D_{R|k}$ is the ring extension of R generated by the partial derivatives $\{\partial_i := \frac{d}{dx_i} \mid i = 1, \ldots, d\}$. In this setting, G. Lyubeznik [3] proved some finiteness properties of local cohomology modules using the fact that they are holonomic. This is a nice class of $D_{R|k}$-modules satisfying some good properties, in particular they have finite length.

When k is a field of characteristic $p > 0$, $D_{R|k}$ is the ring extension of R generated by the set of differential operators $\{\partial_i^{<t>} := \frac{1}{t!}\frac{d^t}{dx_i^t} \mid t \in \mathbb{N} \ , \ i = 1, \ldots, d\}$ so it is no longer a Noetherian ring. Therefore, the theory of $D_{R|k}$-modules in positive characteristic do not behave as in the case of characteristic zero as it was pointed out in [1].

The aim of this talk is to give a better understanding of $D_{R|k}$-modules in positive characteristic. In particular, we are interested in the notion of holonomic modules and its comparison with the category of F-finite F-modules introduced by G. Lyubeznik [4]. The main ingredients we are going to use are the rings of differential operators of level e given by P. Berthelot [2] and the so-called Frobenius descent.

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Algebra and Algebraic Methods. Aula 09
SATURDAY 4. 10:00

Geometric Calculation of the Invariant Integral of Classical Groups

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Abstract

An affine k-group $G = \operatorname{Spec} A$ is semisimple if and only if A^* splits into the form $A^* = k \times B^*$ as k-algebras, where the first projection $\pi_1 \colon A^* \to k$ is the morphism $\pi_1(w) := w(1)$ ([2, Th. 2.6]). The linear form $w_G := (1, 0) \in k \times B^* = A^*$ will be referred to as the *invariant integral* of G.

In the theory of invariants the calculation of the invariant integral w_G is of great interest, because it yields the calculation of the invariants of any representation. The aim of this article is the explicit calculation of w_G when $G = Sl_n, Gl_n, O_n, Sp_{2n}$ ($\operatorname{char} k = 0$) by geometric arguments and by means of the Fourier transform, which is defined below. Although G is not a compact group, it is possible to define the invariant integral of G, the Fourier transform, the convolution product and to prove the Parseval identity , the inversion formula, etc.

Let A_i^* be simple (and finite) k-algebras and let $A^* = \prod_i A_i^*$. On every A_i^*, one has the non-singular trace metric and its associated polarity. Hence, one obtains a morphism of A^*-modules $\phi : A = \oplus_i A_i \hookrightarrow \prod_i A_i^* = A^*$. If $G = \operatorname{Spec} A$ is a semisimple affine k-group and $* : A \to A$, $a \mapsto a^*$ is the morphism induced by the morphism $G \to G$, $g \mapsto g^{-1}$, we prove that ϕ is the morphism

$$A \to A^*,\ a \mapsto w_G(a^* \cdot -)$$

where $w_G(a^* \cdot -)(b) := w_G(a^* \cdot b)$. We will call ϕ the Fourier transform. The product operation on A^* defines, via the Fourier transform, a product on A, which is the *convolution product* in the classical examples.

Let us consider a system of coordinates in G, that is, let us consider $G = \operatorname{Spec} A$ as a closed subgroup of a semigroup of matrices $M_n = \operatorname{Spec} B$. Then A is the quotient of B by the ideal I of the functions of M_n vanishing on G. Hence, A^* is a subalgebra of B^* and

one has that $k \cdot w_G = A^{*G} = \{w \in B^{*G} : w(I) = 0\}$. Moreover, B^G (which is the ring of functions of M_n/G), coincides essentially with B^{*G}, via the Fourier transform. Finally, we prove that given $w \in B^{*G}$, the condition $w(I) = 0$ is equivalent to $w(I^G) = 0$, which is a finite system of equations "in each degree".

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Algebra and Algebraic Methods. Aula 09
FRIDAY 3. 18:30

Supercharacter theories for finite linear p-groups

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Abstract

The group of unipotent uppertriangular matrices with coefficients in a finite field is famous for having a completely intractable character theory. Neither the conjugacy classes, nor the irreducible characters, can be given in any generality. By taking certain unions of conjugacy classes, and sums of irreducible characters, a tractable theory is achieved. The resulting "superclasses" and "supercharacter" are indexed by set partitions and have combinatorics similar to the Young tableaux of the symmetric group. A similar theory is developed for maximal unipotent subgroups of other classical groups (joint work with Ana M. Neto).

Functional Analysis. Aula 10
SATURDAY 4. 12:00

High dimensional sections and projections of convex bodies and the isotropy constant

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Abstract

The asymptotic geometric analysis studies the theory of convex bodies when the dimension goes to infinity. An approach to the concentration of mass, sections, orthogonal projections and the isotropy constant will be explained.

Functional Analysis. Aula 10
SUNDAY 5. 10:00

Banach spaces and harmonic analysis associated with Laguerre and Bessel operators

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Abstract

Vector valued harmonic analysis is closely related to the geometry of Banach spaces. The fact that a certain property, for instance, the boundedness of certain classical operators, is to be true when functions valued in a Banach space are considered, is related to geometrical or topological properties of underlying Banach spaces. Thus, new characterizations of old properties are obtained or new type of Banach spaces appear. Results of Burkholder and Bourgain were crucial in this area. Our objective in this talk is to present new characterizations for well-known properties of Banach spaces (namely, UMD property, martingale type and cotype, Hardy-Litllewood property,...) in terms of the boundedness of certain harmonic analysis operators (Riesz transforms, maximal operators, Littlewood-Paley g-functions,...) associated with Laguerre and Bessel operators.

Functional Analysis. Aula 10
FRIDAY 3. 18:30

Parameter dependence of solutions of differential equations on spaces of distributions

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Abstract

We report on joint work with Pawel Domański. The lecture is based on our papers [1, 2] and on the paper by Domański [3].

We investigate whether for every linear partial differential operator with constant coefficients $P(D) : \mathcal{D}'(\Omega) \to \mathcal{D}'(\Omega)$, $\Omega \subset \mathbb{R}^d$ convex open, and every family of distributions $(f_\lambda)_{\lambda\in U} \subseteq \mathcal{D}'(\Omega)$ depending holomorphically (or smoothly C^∞ or in a real analytic way) on the parameter λ running through an arbitrary Stein manifold U (or C^∞-manifold U or real analytic manifold U), there is an analogous family $(u_\lambda)_{\lambda\in U}$ with the same type of dependence on $\lambda \in U$ such that

$$P(D)u_\lambda = f_\lambda \qquad \forall \lambda \in U. \tag{1}$$

In fact we provide a very efficient and general method for solving (1) for various types of dependence on λ. Our results are based in a deep analysis of the splitting of short exact sequences of PLS-spaces. This is a large class containing all the spaces which appear in the analytic applications of linear functional analysis, like spaces of (ultra)distributions, real analytic functions and holomorphic functions.

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Statistics and Biometry. Salón de Grados
SUNDAY 5. 10:00

Growth and Extinction of Populations and Individuals in Randomly Varying Environments

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Abstract

In a randomly varying environment, the per capita growth rate (abbreviately growth rate) of a population can been described by an "average" rate $g(N)$ (usually dependent on population size N) perturbed by a white noise (as a reasonable approximation to a noise with low correlations). So, with $N = N(t)$ being the population size at time t, we consider the general model

$$\frac{1}{N}\frac{dN}{dt} = g(N) + \sigma\varepsilon(t)$$

where $\sigma > 0$ is the noise intensity and $\varepsilon(t)$ is a standard white noise.

Denoting by $W(t)$ the standard Wiener process, we can write the model in the standard form of a stochastic differential equation (SDE)

$$dN(t) = g(N(t))dt + \sigma N(t)dW(t).$$

These models have been studied in the literature for specific functional forms of the "average" growth rate" g (like, for example, the logistic model $g(N) = r(1N/K)$). Since it is hard to determine the "true" functional form of g, one wonders whether the qualitative results (concerning population extinction or existence of a stationary density) are model robust. We have managed to prove the usual qualitative results for a general function g satisfying only some basic assumptions dictated by biological considerations and some mild technical assumptions (see [1], [2]). From the applied point of view, it was embarrassing that the two main stochastic calculus, Itô and Stratonovich, lead to apparently different qualitative results regarding important issues like population extinction and that led to a controversy in the literature on which calculus is more appropriate to model population growth. We have resolved the controversy (see [3], [4]) by showing that g means different types of "average" growth rate according to the calculus used and the apparent difference was due to the wrong implicit assumption that g represented the same "average". Taking

into account the different meaning of g, there is no difference (qualitative or quantitative) between the two calculi.

The results were then generalized to the case of density-dependent noise intensities $\sigma(N)$ satisfying mild assumptions (see [5], [6]).

We have also recently considered (with Patrcia A. Filipe, see [7]) SDE models for the individual growth from birth to maturity of the size (weight, volume, length, ...) of individual animals (or plants) and we briefly report some results with applications to cattle breeding.

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Statistics and Biometry. Salón de Grados
FRIDAY 3. 19:00

Approximate Aggregation Methods and Applications to Population Dynamics

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Abstract

When modelling biological systems, in particular ecological ones, we usually find very complex systems that we should try to manage to get some insights. A first approach to do this consists in building an abstract model describing the real system in detail. This leads to a family of models involving a very large number of variables. The complexity of the system is included in the model and computer simulation becomes the only available tool to carry out its study. At the other extreme we can find those models avoiding almost every detail in order to be mathematically tractable. These models of ecological communities only deal with a few variables, assuming that the internal structure of the population has no important effect and so can be neglected. This assumption corresponds to an approximation of the total system by means of a reduced one that should be checked. However, in most cases, simplified models are used and few arguments are given to justify them. A tool trying to fill the gap between these two approaches is approximate aggregation.

Approximate aggregation consists in describing some features of the dynamics of a general system in terms of the dynamics of a reduced system governed by a few number of global variables. We think of a hierarchically organized population, thus subdivided into sub-populations, which allows distinguishing two processes of a general nature and whose corresponding time scales are very different from each other. Results will be presented showing that, under quite general conditions, the asymptotic behaviour of the general system can be known in terms of the corresponding behaviour of the reduced system.

We present a review ([1],[2],[3]) of aggregation methods for different kinds of discrete dynamical systems linear and non-linear, deterministic and stochastic.

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Statistics and Biometry. Salón de Grados
SATURDAY 4. 13:00

Use of Generalized Additive Models in studies of association, prediction, and clasification

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Abstract

Publications in many biomedical fields have shown an interest in the application of Generalized Additive Models (GAMs, Hastie and Tibshirani, 1990) since this type of models constitute a good compromise between flexibility and interpretability, while avoiding the curse of the dimensionality. GAMs including interactions can be adapted adequately in biomedical studies of association, prediction, and classification. Based on GAMs, flexible effect curves such as the Odds-Ratio (for association purposes), and Receiver Operating Characteristic (ROC) regression curves (for the purposes of classification and prediction) can be readily obtained. The GAM-based statistical procedures may be extended appropriately to deal with some interesting extensions models, like the additive multi-state model for survival analysis. Finally, all the methods are illustrated with real data arising from various biomedical fields, discussing the necessity of development of user-friendly software, to use this modern statistical methodology in practice.

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Functional Analysis. Aula 10
SATURDAY 4. 12:30

Function spaces and Hausdorff dimension on fractals

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Abstract

We start by surveying a possible approach for function spaces of Besov type on special closed subsets of $\mathbb{R}^n$. Afterwards we recall that, in the case of Sobolev spaces and Besov spaces on $\mathbb{R}^n$, the Hausdorff dimension for the graphs of continuous functions belonging to such spaces has been studied by several authors, and that in the case of Besov spaces the final answer concerning the maximal possibility for the value of the Hausdorff dimension of those graphs was given by F. Roueff in his thesis in 2000. Finally, we report on our results on the corresponding problem of the determination of the maximal Hausdorff dimension of graphs of continuous functions on Besov spaces built themselves over fractals sets like d-sets.

The last part of the talk describes joint work with Abel Carvalho, from Univ. Aveiro.

Functional Analysis. Aula 10
SATURDAY 4. 13:00

Approximation and entropy numbers in Besov spaces of generalized smoothness

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Abstract

Let X and Y be Banach (or quasi-Banach) spaces and let $T \in \mathcal{L}(X,Y)$ be a compact operator. In order to measure "the degree of compactness of T" one can use the asymptotic decay of the sequence of approximation numbers of T or the decay of the sequence of entropy numbers of T. In general, the behaviour of these sequences is quite different. However, as we shall show, under certain assumptions the two sequences behave in the same way. Then we shall use this result to determine the exact asymptotic behaviour of entropy and approximation numbers of the limiting restriction operator $J : B^{s,\psi}_{p,q_1}(\mathbb{R}^d) \to B^s_{p,q_2}(\Omega)$, defined by $J(f) = f|_\Omega$. Here Ω is a non-empty bounded domain in $\mathbb{R}^d$, ψ is an increasing slowly varying function, $0 < p < \infty, 0 < q_1, q_2 < \infty$, $s \in \mathbb{R}$, and $B^{s,\psi}_{p,q_1}(\mathbb{R}^d)$ is the Besov space of generalized smoothness given by the function $t^s\psi(t)$.

The talk is based on a joint paper with Thomas Kühn which is going to appear in the Journal of Approximation Theory

Algebra and Algebraic Methods. Aula 09
SATURDAY 4. 11:00

Connections between real algebra and logic

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Abstract

Logicians have introduced a class of structures, called o-minimal structures, whose geometry includes real algebraic and sub-analytic geometry. Real algebraic geometry is based on real algebra and a fundamental object in this context is the real spectrum of a commutative ring (just like algebraic geometry is based on commutative algebra and the fundamental object is the spectrum of a commutative ring). In o-minimal geometry we have the o-minimal spectrum which is used to develop a sheaf cohomology theory generalizing sheaf cohomology in real algebraic geometry (Delfs) and in sub-analytic geometry (Kashiwara-Schapira).

Algebra and Algebraic Methods. Aula 09
SATURDAY 4. 13:00

Irreducible numerical semigroups

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Abstract

This is a survey of several results obtained with M. B. Branco at the Universidade de Évora, J. I. García-García at the Universidad de Cádiz, J. A. Jiménez-Madrid at the Instituto de Ciencias Matemáticas, and J. C. Rosales at the Universidad de Granada.

A *numerical semigroup* is a submonoid of the monoid of nonnegative integers (under addition) with finite complement in it. A numerical semigroup is *irreducible* if it cannot be expressed as the intersection of two numerical semigroups properly containing it. Every numerical semigroup can be written as a (finite) intersection of irreducible numerical semigroups. Minimal decompositions can be thought in terms of cardinality or redundancy. Both concepts do not coincide, and uniqueness is far from being reached.

As we have mentioned above, the complement of a numerical semigroup in the set of nonnegative integers is finite. So it makes sense to think about the largest integer not belonging to a numerical semigroup. This integer is known as the *Frobenius number*, which became really famous mainly for the lack of a general (algebraic) formula to calculate it for numerical semigroups with more than two minimal generators. Irreducible numerical semigroups can be also characterized as those numerical semigroups maximal (with respect to set inclusion) in the set of numerical semigroups with a fixed Frobenius number. This allows to relate this concept with two well known families of numerical semigroups: symmetric and pseudo-symmetric numerical semigroups. An irreducible numerical semigroup is either symmetric or pseudo-symmetric (depending on the parity of its Frobenius number), and the union of the set of symmetric and the set of pseudo-symmetric numerical semigroups yields that of irreducible numerical semigroups.

Symmetric numerical semigroups earned some popularity due to a result by Kunz that states that the semigroup ring associated to a numerical semigroup is Gorenstein if and only if the numerical semigroup is symmetric (semigroup rings associated to pseudo-symmetric numerical semigroups are Kunz rings). Several subfamilies of the class of symmetric numerical semigroups were also widely studied due to their relevance as examples in Algebraic Geometry. These include that of complete intersections, free (in the sense of Bertin and Carbonne) and telescopic numerical semigroups.

In the set of numerical semigroups with a given Frobenius number, the average of irreducible numerical semigroups is really poor. However, this apparent lack contrasts not

only with the idea that every numerical semigroup is the intersection of irreducible numerical semigroups, but also with the surprising fact that every numerical semigroup is one half of infinitely many symmetric numerical semigroups. By one half of a numerical semigroup, we mean the set of integers that multiplied by two are in the semigroup. This set is again a numerical semigroup. Analogously, one fourth of a numerical semigroup can be defined. Rosales has recently shown that every numerical semigroup is one fourth of a pseudo-symmetric numerical semigroup.

Another amazing property of irreducible numerical semigroups is that every positive integer can be realized as the Frobenius number of an irreducible numerical semigroup with at most four generators.

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Statistics and Biometry. Salón de Grados
SATURDAY 4. 10:00

Statistics of Extremes under Censoring Schemes

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Abstract

Statistics of Extremes deals essentially with the estimation of parameters of extreme events like a *high quantile* situated in the border or even beyond the range of the available data. The most common assumption on any set of univariate data, $(X_1, X_2, \cdots, X_n)$, is to consider that we are in the presence of a *complete* sample of size n, either indepently, identically distributed or weakly dependent and stationary, from an unknown distribution function $F = F_X$. In all applications of extreme value theory, the estimation of the *extreme value index* is of primordial importance. Such a parameter measures the heaviness of the tail and has been widely studied in the literature. It is indeed the real parameter γ in the extreme value distribution function $EV_\gamma(x) = \exp(-(1+\gamma x)^{-1/\gamma})$, $1+\gamma x > 0$. This distribution function appears as the limiting distribution of the maximum, linearly normalized, whenever such a non-degenerate limit does exist. For heavy tails, i.e. whenever $\gamma > 0$, we mention the classical *Hill* estimator (Hill, 1975) and one of the most recent *minimum-variance reduced-bias* (MVRB) estimators of the extreme value index (Caeiro et al., 2005) and of extreme quantiles (Gomes and Pestana, 2007). For a general extreme value index estimation, we mention the *moment* (Dekkers et al., 1989), the "*maximum likelihood*" (Smith, 1987; Drees et al., 2004), the *generalized Hill* (Beirlant et al., 1996) and the *mixed moment* (Fraga Alves et al., 2007) estimators. In all these papers the available sample is complete. However, in the analysis of lifetime data or reliability data, observations are usually censored. For simplicity we shall assume first the case of right censorship, where no difficulties appear, following immediately to the case of random censorship, where apart from a recent paper by Einmahl et al. (2008), there is only, as far as we know, a brief reference to the topic in Reiss and Thomas (1997, Section 6.1) and a paper by Beirlant et al. (2007). We shall give here special attention to the estimation of γ, as well as associated high quantile and right endpoint estimation under random censoring, i.e. we shall assume that there is a random variable Y such that only $Z = X \wedge Y$ and $\delta = I_{\{X \le Y\}}$ are observed. The indicator variable δ determines whether X has been censored or not. Consequently, we have access to the random sample (Z_i, δ_i), $1 \le i \le n$, of independent copies of (Z, δ), but our goal is to make inference on the right tail of the unknown lifetime distribution, i.e. on $\overline{F}_X(x) := P(X > x) = 1 - F_X(x)$, while F_Y, the distribution function of Y, is considered to be a nonparametric nuisance parameter. As mentioned in Einmahl *et al.* (2008), all the extreme value index estimators need to be

slightly modified in order to be consistent for the estimation of γ. A possible and simple modification is suggested in Einmahl *et al.* (2008): replace $\widehat{\gamma}(k|Z)$ by $\widehat{\gamma}(k|X) = \widehat{\gamma}(k|Z)/\widehat{p}$, with $\widehat{p} = \frac{1}{k}\sum_{j=1}^{k} \delta_{[n-j+1]}$, with $\delta_{[j]}$ the concomitant value of δ associated with $Z_{j:n}$, the j-th ascending order statistic, $1 \leq j \leq n$, associated with the observed sample $(Z_1, Z_2, \cdots, Z_n)$. We shall here apply the methodology in Einmahl *et al.* (2008) to a few sets of survival data, available in the literature, as well as simulated data, providing some hints for the adequate estimation of the extreme value index, high quantiles and right endpoint of X.

Keywords and phrases. Extreme value index, censoring, extreme quantiles

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Functional Analysis. Aula 10
SUNDAY 5. 11:00

Asymptotics of Toeplitz matrices: recent results

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Abstract

We discuss some recent results on asymptotics of Toeplitz matrices. We consider variations of the classical Szeg-Widom limit theorem for several classes of generating matrix functions. If the smoothness of a generating matrix function of a Toeplitz matrix is less than optimal, then new correcting terms in asymptotic formulas appear. On the other hand, if the smoothness of a generating matrix function is greater than optimal, then the speed of convergence in asymptotic formulas can be specified. For very smooth generating matrix functions remainders in asymptotic formulas go to zero very rapidly. Bounded Wiener-Hopf factorization in decomposing algebras of possibly discontinuous functions plays an essential role in our proofs.

Algebra and Algebraic Methods. Aula 09
SUNDAY 5, 11:00

Regular and biregular smash products

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Abstract

In 1936, John von Neumann introduced a regularity conditions on rings which he subsequently used in his study of continuous geometry. Group rings which are regular in the sense of von Neumann were completely characterised by Auslander, McLaughlin and Villamayor in the 1950's. Alfaro, Ara and del Rio considered regular skew group rings in 1995. In this talk we will discuss von Neumann's regularity condition for smash products of Hopf algebra actions. We shall also discuss biregularity in the sense of Kaplansky and Arens (Transactions AMS, 1948).

Statistics and Biometry. Salón de Grados
SUNDAY 5. 11:00

Optimal designs for models with potential censoring

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Abstract

This presentation deals with optimal design theory for models with potential censoring either on independent or dependent variables. On the one hand optimal approximate designs when an independent variable might be censored are considered. The problem is which design should be applied to obtain an optimal approximate design when the censored distribution function is assumed known in advance. The approach for finite and continuous design spaces deserves different attention. In both cases equivalent theorems and algorithms are provided in order to calculate optimal designs. Some examples illustrate this approach for D–optimality.

On the other hand a development of the optimal design theory is carried out for a particular Cox Regression problem. The failure time is modelized according to a probability distribution depending on some explanatory variables through a linear model. At the end of the study some units will have not failed and thus their time records will be censored. In order to deal with this problem from an experimental design point of view it will be necessary to assume a probability distribution of the time of debut of an experimental unit in the study. Then an optimal conditional design will be computed at the beginning of the study for any possible given time of debut. Thus, every time a new unit enters the study there is an experimental design to be applied. A particular and simple case is used throughout the presentation in order to illustrate the procedure.

keywords: Censoring distribution, D–optimality, Information Matrix, Marginally restricted designs, Survival analysis.

Functional Analysis. Aula 10
SATURDAY 4. 10:00

Bishop-Phelps Theorem for operators

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Abstract

We report on joint work with María Acosta, Richard Aron, and Domingo García. Our talk is based on our papers [1, 2].

The classical Bishop-Phelps theorem (1961) states that for any Banach space X the set of linear forms that that attain their norms is always dense in X^*. This was improved by Bollobas (1970) showing that the points in which the norms are attained can be controlled very nicely. On the other hand Lindenstrauss (1963) proved that for certain Banach spaces X and Y, the subset of norm attaining operators from X into Y is not norm dense in the space of all continuous and linear operators $L(X, Y)$. In this talk we report on recent work about when it is possible to give Bishop-Phelps-Bollobas type results for operators and also about a general Lindenstrauss type theorem: The set of multilinear mappings on Banach spaces such that all their Arens extensions attain the norm are dense with the supremum norm in the space of all continuous multilinear mappings.

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Algebra and Algebraic Methods. Aula 09
SATURDAY 4. 12:30

On finite complete rewriting systems and completely 0-simple semigroups

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Abstract

Given a binary relation R on a free semigroup over an alphabet A we can associate to it a (string-)rewriting system by considering the single-step reduction $\rightarrow_R$. At the same time we can associate to R the Thue congruence $\overset{*}{\leftrightarrow}_R$ generated by R, and consequently the semigroup resultant of the quotient of A^+ by $\overset{*}{\leftrightarrow}_R$. Regard R as a rewriting system and study the properties of the single-step reduction corresponds to a Theoretical Computer Science point of view, while the study of the semigroup quotient $A^+/\overset{*}{\leftrightarrow}_R$ corresponds to an Algebraic point of view.

The study of the properties of the single-step reduction, namely finiteness and completeness (that is, noetherian and confluent), can be used to solve word problems among other algebraic decision problems. On the other hand completely 0-simple semigroups are one the most important theoretical semigroup constructions, since it is known by the structure theorem of Rees that every completely 0-simple semigroup is isomorphic to some $I \times \Lambda$ Rees matrix semigroup $\mathcal{M}^0[G; I, \Lambda; P]$ over a group G.

In this talk we intent to briefly discuss the above notions and present results relating the mentioned properties of a rewriting system associated to the group G with the same properties on the completely 0-simple semigroup $\mathcal{M}^0[G; I, \Lambda; P]$.

Statistics and Biometry. Salón de Grados
SATURDAY 4. 11:00

Inference in multi-state survival data

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Abstract

In longitudinal studies of disease, patients can experience several events through a follow-up period. In these studies, the sequentially ordered events (gap times) are often of interest. The events of concern may be of the same nature (e.g. cancer patients may experience recurrent disease episodes) or represent different states in the disease process (e.g. alive and disease-free, alive with recurrence and dead). If the events are of the same nature this are usually referred as recurrent event, whereas if they represent different states (i.e. multi-state models) they are usually modelled thought their intensity functions. In this talk we present new estimators for several quantities in a three-state model. We present a simple estimator for the bivariate distribution function for censored gap times and estimators for the transition probabilities. Another topic of discussion is the relationship between the different covariates and disease evolution. The proposed methods are applied to a database on breast cancer.

Statistics and Biometry. Salón de Grados
SATURDAY 4. 12:00

Ensuring the additivity property for estimating the total tree biomass

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Abstract

The estimation of tree biomass is a very important issue for both practical forestry and scientific purposes. As forestry activities mainly affect the exchange of carbon dioxide between the land and the atmosphere, the management of these activities offers an opportunity to isolate atmospheric carbon that constitutes a portion of a regions greenhouse gas contribution. Then, estimation of aboveground tree biomass is an essential aspect of carbon stocks studies and the effects of deforestation and carbon sequestration in the global carbon balance. The most common procedure for estimating tree biomass is the use of direct measurements of biomass and regression techniques. In forestry research, the development of allometric equations or models relating the weight of the different parts of the tree with other quantities, usually the stem diameter, is essential to predict the tree biomass. However a major drawback arises because the predictions of the weight, obtained for the different tree components, do not add up to the predictions derived from modelling the total tree weight, that is, the property of biomass additivity is not fulfilled. This means that the regression functions for the different tree components are not consistent with each other. The main goal of this work is to propose and compare alternative techniques to ensure additivity of the regression predictions providing some advantages over the widely used logarithmic model. Firstly, non linear seemingly unrelated equations are derived, because this model accounts for statistical dependencies among the data. Secondly, we consider the application of segmented regression because an apparent non linear relationship between two variables can be split up into two or more linear regressions. The procedure is very appealing because it can offer a simpler and clearer interpretation than more complex models. However, if the break point of the segmented regression is not the same for all the tree components, the

additivity property is not fulfilled. Then, we propose the calculation of a common break point for all the tree components such that the desired additivity property is attained. Finally, as the distribution of the weights is asymmetric, we also propose to fit quantile segmented regression in order to provide a more complete picture of the whole distribution. The procedures are illustrated with a data set of 42 beech trees (Fagus Sylvatica) from a forest in the northern part of Navarra, Spain.

Algebra and Algebraic Methods. Aula 09
SUNDAY 5, 10:30

Polynomial identities in Jordan systems

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Abstract

Polynomial identities play a fundamental role in nonassociative theory. In this talk we review some of the results of PI-theory in Jordan systems, as well as some of its structural applications.

Statistics and Biometry. Salón de Grados
SUNDAY 5. 10:30

Bayesian analysis of allelic penetrance models for complex binary traits

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This is a joint work with Nuno Sepúlveda and Carlos Penha-Gonçalves, Instituto Gulbenkian de Ciência.

Abstract

Complex binary traits result from an intricate network of genetic and environmental factors. To aid their genetic dissection, several generalized linear models have been used to detect interaction between genes. However, it is recognized that these models have limited genetic interpretation. As an attempt to overcome this problem, we have previously proposed the allelic penetrance approach to model dominance and recessiveness at a single locus and to describe independent, inhibition and cumulative actions between two diallelic loci. Classically, recessive inheritance requires the expression of both recessive alleles at homozygotes to obtain the phenotype (type I recessiveness). In previous work, we alternatively define recessiveness as a situation where a recessive allele could express the phenotype when the dominant allele is not active (type II recessiveness).

Here the allelic penetrance models are revisited under both definitions of recessiveness. We apply Bayesian methods to analyze two data sets: one regarding the effect of a given haplotype on the inheritance of two immunoglobulin deficiencies (IgD and IgG4) in humans and the other related to joint action of two loci on Listeria infection susceptibility in mice resulting from an intercross between two strains. Our results indicate that IgD and IgG4 deficiencies are dominant and type I recessive traits with respect to the given haplotype, respectively. Regarding Listeria infection, susceptibility appears to be controlled by an independent action between a locus with a dominant allele inherited from the resistant strain and a locus at another chromosome with a type I recessive allele derived from the susceptible strain.

Algebra and Algebraic Methods. Aula 09
FRIDAY 3. 19:00

The Cuntz semigroup and its impact into classification

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Abstract

In 1976, Elliott, building on work of Glimm and Bratteli, gave a classification of countable limits of semisimple algebras over algebraically closed fields. This in fact applies to classify, in terms of ordered K-Theory, all approximately finite dimensional C*-algebras ([3]). His discovery led him to conjecture, around 1989, that a wide class of simple C*-algebras would be classified by K-theoretic invariants, and bolstered his claim by showing with D. Evans that the algebras known as irrational rotation algebras were so classified ([4]).

The so-called Elliott Programme has enjoyed a resurgence of late, owing to the discovery by M. Rørdam ([7]) and later by A. S. Toms ([8]) that K-Theory does not suffice for the classification of all separable, simple, amenable C*-algebras. There are two ways forward: restrict the class of algebras considered or enlarge the proposed invariant. Both of these courses have been pursued vigorously over the past three or four years, leading to several breakthroughs in Elliott's Programme.

The following question has received a lot of attention recently: "Can one characterize the largest class of simple, separable, amenable C*-algebras for which Elliott's original conjecture holds? " Two properties related to this stand out: $\mathcal{Z}$-stability and strict comparison of positive elements.

Here, $\mathcal{Z}$ is the so-called *Jiang-Su algebra* ([5]). The interest of the first property stems from the fact that taking a tensor product with $\mathcal{Z}$ is inert at the level of K-Theory. The second property of interest is related to an object called the Cuntz semigroup. This can be associated to any C*-algebra and its recent popularity is due to its extreme sensitivity as an invariant. This is due to the fact that it is able to distinguish simple, separable, amenable C*-algebras with the same K-Theory and tracial space. The property of strict comparison says, roughly, that the natural partial order on the Cuntz semigroup is determined by certain states. This property often – conjecturally, always – implies that the Cuntz semigroup can be recovered functorially from K-Theory and traces (see [1], [2], [6]).

The purpose of the talk will be to give an account of these recent goings-on.

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Functional Analysis. Aula 10
SATURDAY 4. 10:30

On some inverse problems in the theory of orthogonal polynomials and their connections with Jacobi operators

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Abstract

Given a positive Borel measure whose support is an infinite set of points in the real line the problem of the construction of an orthogonal polynomial sequence (OPS) with respect to that measure is known as *direct problem*. An *inverse problem* (IP) consists in finding the orthogonality measure when only some information is known about a given OPS (e.g., via a three-term recurrence relation, or via the knowledge of some algebraic or analytical relations between the given OPS and some other ones). In this talk we will consider several models of inverse problems in the Theory of Orthogonal Polynomials. For instance:

(IP) Let $(p_n)_n$ be a given monic OPS and k a fixed positive integer number such that $k \geq 2$. We want to analyze conditions under which this OPS is constructed from a polynomial mapping in the following sense: to find another monic OPS $(q_n)_n$ and two polynomials π_k and θ_m, with degrees k and m (resp.), with $0 \leq m \leq k-1$, such that

$$p_{nk+m}(x) = \theta_m(x) q_n(\pi_k(x))$$

for all $n = 0, 1, 2, \cdots$.

Assuming that the support of the positive Borel measure with respect to which $(p_n)_n \geq 0$ is orthogonal is a compact set, we give explicitly this measure in terms of the orthogonality measure for the OPS $(q_n)_n$. One gets orthogonality on several intervals of the real line, with possible mass points located at the gaps between these intervals. A connection with the so-called sieved OPS on the unit circle is made, leading to families of OPS on the unit circle with an orthogonality measure supported on several arcs of the unit circle. Our results enable us to recover several known results in the literature, including some ones which arise from connections between this kind of transformation laws and the spectral theory of self-adjoint Jacobi operators.

Statistics and Biometry. Salón de Grados
FRIDAY 3. 18:30

On a family of circular distributions due to Batschelet

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Abstract

Circular data arise in scientific disciplines as diverse as meteorology, the earth sciences, biology, medicine and the political sciences. For instance, they might represent the directions of prevailing winds in the vicinity of a wind farm, the orientations of fault lines in geological bedrock, the directions of migrating birds, the degree of flexibility of the legs of injured cyclists, or the times of violent attacks in occupied Iraq.

In recent years, various families of symmetric unimodal distributions have been proposed in the literature as models for circular data. Three particularly flexible ones are the wrapped symmetric stable ([2], Section 2.2.8), wrapped t ([4]) and the Jones–Pewsey ([3]) families. Each of these families has three parameters controlling the distribution's mean direction, concentration/scale, and shape.

Here, consideration is given to an alternative three-parameter family of symmetric unimodal circular distributions originally proposed by Batschelet ([1], p. 288) as an extension to the von Mises distribution. Batschelet's family is an interesting one as it not only contains sharply peaked distributions, like the three families already mentioned, but also distributions which are far more flat-topped than the von Mises. The family's fundamental properties are derived, and an in-depth comparison made between it and its direct competitors. Likelihood based techniques are developed which can be used to conduct point and set estimation for the parameters as well as hypothesis testing. Finally, an analysis of a data set taken from the geological literature is presented which illustrates how the family and its direct competitors can be employed in the search for parsimonious models for circular data.

This is joint work with Kunio Shimizu of Keio University.

Key words and phrases: Flat-topped distributions; likelihood based inference; sharply peaked distributions; symmetrical unimodal distributions; von Mises distribution; wrapped symmetric stable family.

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Functional Analysis. Aula 10
FRIDAY 3. 19:00

Cuntz-Krieger sub-algebras and orbit representations

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Abstract

Given a Cuntz-Krieger algebra $\mathcal{O}_A$ we build a Markov map f, defined in the interval $I = [0, 1]$, whose transition matrix is A. The orbits of the discrete dynamical system (Ω, f), where $\Omega \subset I$ is the minimal invariant set under iteration of f, gives us a family of representations of $\mathcal{O}_A$. In our work we go further and analyze the conditions in which we obtain orbit representations of Cuntz-Krieger algebras $\mathcal{O}_{A'}$ arising from the iterates f^k defined on some appropriate Cantor set $\Omega' \subset \Omega$. In this case these Cuntz-Krieger algebras $\mathcal{O}_{A'}$ are sub-algebras of the original algebra $\mathcal{O}_A$.

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Algebra and Algebraic Methods. Aula 09
SATURDAY 4. 10:30

On certain families of partial exponential sums

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Abstract

Partial exponential sums are a generalization of classical exponential sums which are useful, for instance, for studying the distribution of the rational points along the fibers of a morphism between varieties over a finite field. They share some properties with the classical ones: their values are also governed by a certain L-function, which is rational and whose reciprocal roots are pure Weil algebraic integers. However, their cohomological interpretation is much more complicated in general. It has a strong dependence on the geometry of the morphism and we only have good bounds for particularly nice settings. In this talk we will focus on one particularly simple family, which illustrates the different methods used to study this kind of sums and the results that one would expect in general. It is based on joint work with Daqing Wan.

Statistics and Biometry. Salón de Grados
SATURDAY 4. 10:30

Competition models and other probabilistic cellular automata

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Abstract

Consider M distinct species living in the space $\mathbf{Z}$ ($\mathbf{Z^d}$). Each site of the space can be occupied by only one individual. At each time step, each individual produces one offspring that goes to colonize a neighboring site, killing the old particle that lived in that site and competing against the offspring of other individuals. Is there an equilibrium measure? Is there, in long-term, coexistence of different species?

This competition model and other biological or physical models can be seen as Probabilistic Cellular Automata (PCA).

Probabilistic Cellular Automata (PCA) are discrete time stochastic processes with state space $\mathbf{X} := W^{\mathbf{Z^d}}$, where $W = \{1, \ldots, M\}$ and $d \geq 1$, with finite range interactions on the integer lattice $\mathbf{Z^d}$, that is, at each point $z \in \mathbf{Z^d}$ there is a particle (individual) which takes values in the set W and that changes its value according some transition rule which depends on the values of its neighbor particles.

Both in continuous and discrete time processes, duality is a powerful tool for studying these systems ([1]), since it provides relevant information about the evolution of the process under consideration from the study of other simpler process, the dual process.

In this work, we introduce a new form of duality for multistate PCA. This new idea of duality includes the classical duality and allows us to study processes that do not have a dual in the classical sense.

Using this duality, ergodicity results for wide classes of PCA are obtained. The results provide the existence of an equilibrium measure for the competition model described above and for many other models as multi-opinion models or Domany-Kinzel models, solving some open problems in the literature ([2]).

This is a joint work with F.J. López and M. Sobottka.

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Algebra and Algebraic Methods. Aula 09
SATURDAY 4. 12:00

Conjugacy in virtually free groups

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Abstract

Moldavanskii proved in 1969 that, given finitely generated subgroups $H_1, \ldots, H_n$, $K_1, \ldots, K_n$ of a free group F, it is decidable whether or not there exists some $x \in F$ such that $xH_ix^{-1} = K_i$ for $i = 1, \ldots, n$. As a consequence from the results of Diekert, Gutiérrez and Hagenah (2005) the above problem remains decidable with constraints of type $x \in L$, where L denotes an arbitrary rational subset of F. Making use of automata combinatorics and the dynamical study of the continuous extensions of automorphisms to the boundary of the free group, we can compute the solution set of the equation $x^{-1}\varphi(x) \in L$ for $L \subseteq F$ rational and $\varphi \in \mathrm{Aut}F$ virtually inner. One of the consequences is the generalization of Moldavanskii's Theorem to virtually free groups, inclusive with constraints that go beyond context-free languages.

These results were obtained in collaboration with Manuel Ladra (University of Santiago de Compostela).

Functional Analysis. Aula 10
SUNDAY 5. 10:30

Operator factorization, matrix identities and normalization

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Abstract

This is a short survey on some aspects of operator relations given by factorization or operator matrix identities. First we expose the basic concepts and discuss properties of - and equivalence between - various operator relations. Secondly we present applications to linear boundary value problems (for the Helmholtz equation in conical domains) and to classes of singular equations (of Toeplitz or Wiener-Hopf plus Hankel type) which lead to concrete results such as Fredholm criteria for classes of singular operators, the analytical solution of boundary value problems and normalization of ill-posed problems.

Statistics and Biometry. Salón de Grados
FRIDAY 3. 18:00

Location-scale invariant Bickel-Rosenblatt tests and their application to assessing multivariate normality

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Abstract

Location-scale invariant Bickel-Rosenblatt goodness-of-fit tests (IBR tests) are considered in this talk to test the hypothesis that f, the common density function of the observed independent d-dimensional random vectors, belongs to a null location-scale family of density functions. The asymptotic behaviour of the test procedures for fixed and non-fixed bandwidths is studied by using an unifying approach. We establish the limiting null distribution of the test statistics, the consistency of the associated tests and we derive its asymptotic power against sequences of local alternatives. These results show the asymptotic superiority, for fixed and local alternatives, of IBR tests with fixed bandwidth over IBR tests with non-fixed bandwidth. From a finite sample point of view the behaviour of the IBR tests strongly depends on the choice of a smoothing parameter h (bandwidth). In the important case of assessing multivariate normality we give a theoretical and finite sample based description of the role played by the smoothing parameter in the detection of departures from the null hypothesis and we propose an easy to use rule for choosing h that produces a test with the omnibus property.

Algebra and Algebraic Methods. Aula 09
FRIDAY 3. 18:00

Classifying saturated fusion systems over 2-groups

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Abstract

The notions of abstract fusion systems and linking systems over a p-group S generalize those associated to a finite group, and have importat applications in Algebraic Topology and Respresentation Theory. In the talk, we will give an overview of the relevant concepts, including the notions of critical and F-essential subgroups, which play a crutial role in defining fusion systems. Then we will present a systematic procedure to find those subgroups of S and how to determine all nonconstrained centerfree fusion systems over S. We will also apply these methods to some examples.

This is joint work with Bob Oliver (Universitè Paris 13).

Functional Analysis. Aula 10
SATURDAY 4. 11:00

Functional Analysis in Quantum Information: some results

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Abstract

The fast growing area of Quantum Information is demanding colaboration of different scientific disciplines. I will present some results of Functional Analysis and their application to the solution of relevant questions in QI. Most of the talk will be based on [1] and related work.

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Statistics and Biometry. Salón de Grados
SATURDAY 4. 12:30

Dynamical analysis and design of biochemical kinetic networks

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Abstract

Systems Biology is an emerging field that uses a global and integrative perspective to capture and understand the behavior of complex living organisms. The study of metabolic networks, the set of chemical processes and reactions occurring in a cell, represents nowadays a topic of intense interest in the scientific community, a fact which is a reflection of its expected impact on several areas such as biotechnology and medicine. The complete quantitative dynamic characterization of biochemical networks remains an open problem, due to their intrinsic non-linearity and high number of parameters to be inferred. A multidisciplinary approach, involving the optimal integration of mathematical formulation and experimental data, is especially promising. In particular, one major challenge is to estimate the parameters of the systems of non-linear differential equations that better adjust experimental multivariate time-series of metabolite concentrations. This step constitutes a bottleneck of the modeling procedure; due to the existence of local minima, there is no straightforward procedure that guarantees convergence to a unique, global solution. Other problems include the change of the network graph through gene mutations in order to maximize fluxes and optimize the production of desired products. Project DynaMo is a three-year project that will address some of these problems and will involve a multidisciplinary team with several partners and collaborations.

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Posters Sessions

II Iberian Mathematical Meeting
Badajoz, 3-5 October, 2008

Functional Analysis
FRIDAY 3, 16:00-19:30. Poster 1.

Chord in the sphere, middle points and inner product spaces

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Abstract

Let X be a real normed space with unit sphere S.

It is interesting to find simple properties that characterize when the norm in X is induced by a inner product or, in other words, when the unit sphere of its two-dimensional subspaces is an ellipse [1].

We prove, [2],[3], that this happens when every chord of S that supports the homothetic sphere of radious $\frac{1}{2}$, onwards $\frac{1}{2}S$, touches $\frac{1}{2}S$ at its middle point or, equivalent, in terms of the norm:

$$u, v \in S, \ \inf_{t\in[0,1]} \|tu + (1-t)v\| = \frac{1}{2} \ \Rightarrow \ \frac{u+v}{2} \in \frac{1}{2}S$$

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Algebra and Algebraic Methods
FRIDAY 3, 16:00-19:30. Poster 2.

Modules, ideals and their Rees algebras

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Abstract

In studying Rees algebras of modules one may attach to a module several classes of ideals, that carry important infomation about the structure and properties of the module. The Fitting and Bourbaky ideals are particular examples of such classes. In this poster we shall emphasize the relations between them and their importance in the study of the Rees algebras of modules.

References

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Statistics and Biometry
FRIDAY 3, 16:00–19:30. Poster 3.

Partial sufficiency and density estimation

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Abstract

Several works by Wertz explore the relationships between the density estimation problem and the concepts of sufficiency [2], unbiasedness [**?**], invariance [4] and quasi-invariance [5]. Here we will extend these studies to cover the relationship with the concept of partial sufficiency, in the sense of [1]. Namely, we study the usefulness of the concept of partial sufficiency in nonparametric density estimation.

Let $(\Omega_i, \mathcal{A}_i, \mu_i)$, $i = 1, 2$, be σ-finite measure spaces, λ another σ-finite measure on $(\Omega_1, \mathcal{A}_1)$, $(\Omega, \mathcal{A}) = (\Omega_1, \mathcal{A}_1) \times (\Omega_2, \mathcal{A}_2)$, $\mathcal{L}$ a family of measurable nonnegative functions $\ell : (\Omega, \mathcal{A}) \to [0, +\infty)$ such that $\int_{\Omega_2} \ell(\omega_1, \omega_2) d\mu_2(\omega_2) = 1$, for every $\omega_1 \in \Omega_1$, $\mathcal{B}$ a countably generated sub-σ-field of $\mathcal{A}^n$, and Δ_1 a family of probability densities with respect to μ_1. We denote $\Delta := \{f \otimes \ell \colon f \in \Delta_1, \ell \in \mathcal{L}\}$ where $(f \otimes \ell)(\omega_1, \omega_2) = f(\omega_1)\ell(\omega_1, \omega_2)$ is a probability density on $\mathcal{A}$ with respect to $\mu_1 \times \mu_2$.

Our purpose is to estimate the density $f \in \Delta_1$ from a sample of size n taken from $f \otimes \ell$, leaving ℓ as a nuisance parameter. Any measurable function $f_n : (\Omega_1 \times \Omega^n, \mathcal{A}_1 \times \mathcal{A}^n) \to \mathbb{R}$ will be called an estimator of f. $\Lambda_p(\mathcal{A}_1 \times \mathcal{A}^n)$ stand for the class of all $(\mathcal{A}_1 \times \mathcal{A}^n)$-measurable density estimators f_n for which $\|f_n\|_p := \left\{\mathbb{E}_f \int |f_n(\omega_1, \bar{\omega}) - f(\omega_1)|^p d\lambda(\omega_1)\right\}^{1/p} < \infty$.

Theorem: *Suppose that, for every $\ell \in \mathcal{L}$, we have $\sup_{f \in \Delta_1} \int (f \otimes \ell)^p d\lambda < \infty$. If $\mathcal{B}$ is partially sufficient for f then, for every estimator $f_n \in \Lambda_p(\mathcal{A}_1 \times \mathcal{A}^n)$ which is ancillary for the parameter ℓ, there exists $f_n^* \in \Lambda_p(\mathcal{A}_1 \times \mathcal{B})$ such that $ML_pE(f_n^*, f) \leq ML_pE(f_n, f)$, where*

$$ML_pE(f_n, f) := \left\{\mathbb{E}_f \int |f_n(\omega_1, \bar{\omega}) - f(\omega_1)|^p d\lambda(\omega_1)\right\}^{1/p}.$$

Moreover, f_n^* can be chosen in a way such that $f_n^*(\omega_1, \cdot)$ be a version of the conditional expectation $\mathbb{E}_{f\otimes\ell}[f_n(\omega_1, \cdot)|\mathcal{B}]$, λ-a.e.; that is, there is a λ-null set $N_1 \in \mathcal{A}_1$ such that

$$\int_B f_n^*(\omega_1, \bar{\omega}) dP^n_{f\otimes\ell}(\bar{\omega}) = \int_B f_n(\omega_1, \bar{\omega}) dP^n_{f\otimes\ell}(\bar{\omega}), \quad \forall B \in \mathcal{B}, \forall f \otimes \ell \in \Delta, \forall \omega_1 \notin N_1.$$

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Functional Analysis
FRIDAY 3, 16:00–19:30. Poster 4.

C^*-algebras of Singular Integral Operators with Shifts Having a Nonempty Set of Fixed Points

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Joint with
M. A. Bastos and Yu. I. Karlovich

Abstract

The C^*-subalgebra $\mathfrak{B} = \operatorname{alg}(PSO(\mathbb{T}), S_{\mathbb{T}}, U_G)$ of $\mathcal{B}(L^2(\mathbb{T}))$ generated by all multiplication operators aI by slowly oscillating and piecewise continuous functions, by the Cauchy singular integral operator $S_{\mathbb{T}}$ and by the range of a unitary representation of an amenable group G of diffeomorphisms $g : \mathbb{T} \to \mathbb{T}$ with any nonempty set Λ of common fixed points is studied. A symbol calculus for the C^*-algebra $\mathfrak{B}$ and a Fredholm criterion for its elements are obtained. For the C^*-algebra $\mathcal{A} = \operatorname{alg}(PSO(\mathbb{T}), U_G)$ composed by all functional operators in $\mathfrak{B}$, an invertibility criterion for its elements is also established. Both the C^*-algebras $\mathfrak{B}$ and $\mathcal{A}$ are investigated by using a generalization of the local-trajectory method for C^*-algebras associated with C^*-dynamical systems which is based on the notion of spectral measure, and developed for the case when the action of an amenable discrete group G on the maximal ideal space $M(\mathcal{Z})$ of a central C^*-algebra $\mathcal{Z} \subset \mathfrak{A}$ is not topologically free [**?**].

To investigate the C^*-algebra $\mathcal{A}$ and the quotient C^*-algebra $\mathfrak{B}^\pi := \mathfrak{B}/\mathcal{K}$, where $\mathcal{K} := \mathcal{K}(L^2(\mathbb{T}))$ is the ideal of all compact operators in $\mathcal{B}(L^2(\mathbb{T}))$, we decompose these C^*-algebras in orthogonal sums of operator C^*-algebras obtained with the help of spectral projections related to G-invariant subsets of the maximal ideal space of appropriate commutative C^*-subalgebras of $\mathcal{A}$ and $\mathfrak{B}^\pi$, respectively, and we investigate the invertibility in each one of these new C^*-algebras [**?**].

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Statistics and Biometry
FRIDAY 3, 16:00–19:30. Poster 5.

Weighted Conditional Least Squares Estimation in Bisexual Branching Processes with Immigration

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Abstract

The Bisexual Galton–Watson branching process (BGWP), introduced by Daley (1968), is a discrete time branching model that is well-suited to describing the probabilistic evolution of populations where females and males coexist and form couples (mating units) which reproduce independently with the same offspring probability distribution. To describe the probabilistic evolution of more complicated populations with sexual reproduction, there have been introduced modified bisexual branching models. An important special case of these processes are the BGWPs with immigration which have been introduced in González, Molina and Mota (1999). In particular, in this work we will be concerned with BGWPs which allow in each generation the immigration of females and males. The probabilistic theory about this model has been studied in González, Molina and Mota (2000, 2002).

There exists a plentiful literature on the topic of estimation of parameters associated with asexual Galton-Watson processes with immigration (GWPI) (see Wei and Winnicki (1990) and references within it). It is worth pointing out the approach achieved in Wei and Winnicki (1990) in the attempt to solve the problem of providing estimators of the parameters of a GWPI which do not required any prior knowledge about the growth behaviour of the process. These authors proposed a unified estimation theory based on conditional weighted least squares theory. However, there are no studies about the estimation problems arising from the BGWPs with immigration. Motivated by the work of Wei and Winnicki (1990), in this communication we will propose conditional weighted least squares estimators of the offspring and immigration mean vectors of a BGWP with immigration and we will derive their asymptotic properties when the process is subcritical, focussing our attention on their strong consistency and limit distributions.

Acknowledgement

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Algebra and Algebraic Methods
FRIDAY 3, 16:00-19:30. Poster 6.

An application of Lie Theory to deal with some problems in Economics

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Abstract

It is well-known the use of Lie Theory in the solving of problems referred to other sciences different from Mathematics, like Physics and Engineering, for instance. However, all of them are usually experimental or technical; being nowadays more unknown the applications of this theory to other non-experimental, non-technical sciences. In this sense, this poster is devoted to show some applications of Lie Theory to Economics. Such applications are only known in some specific economical topics.

Regarding the relation between Lie Theory and some economical problems, we would like to point out that there are several recent works, all of them within this new century, which consistently represent the advances obtained by applying well-known properties of Lie groups and algebras to several economical concepts. So Lo and Hui presented in [8] and [9] (at the beginnings of 2000s) different techniques based on Lie algebras to deal with the price of financial derivatives and with the called *multi-asset financial derivatives*. In other works ([6] and [7]), they apply similar techniques to evaluate the options of *moving barrier*. Independently, Björk and Landén (2002) studied in [3], by using Lie algebras, some models of *interest-rate*, previously introduced by Björk himself in [2]. Polidoro (2003) studied a financial problem by using nilpotent Lie groups in [10]. Basov (2004) described some methods based on Lie groups in order to solve the multidimensional *screening problem* in [1]. Besides, Gaspar [5] have studied a general model for the structure of *forward prices*,

by using the methodology of Lie algebras by Björk. Finally, Björk himself have studied some applications of Lie algebras to certain economical concepts like *constant volatility* and *constant directional volatility*.

In this poster we briefly comment first the basic aspects in some of the works previously cited, in order to get a global view of the current status in the application of Lie Theory to Economics. In the second place, we expose a critical study and the advances and main results obtained by ourselves when considering this innovative research line, focusing it in the concept of holotheticity, introduced by Sato in [11]. Taking into account all that has been previously commented, we are dealing with some open economical problems by considering Lie Theory to progress in their study. Previously, some of them have been already analyzed and dealt by one of the authors in [4]. More concretely, the mathematical foundations for the concept of holotheticity are explained and checked in order to prove that the definition and some properties given by Sato are mathematically correct.

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Functional Analysis
FRIDAY 3, 16:00-19:30. Poster 7.

Interval neutrosophic sets and topology

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Abstract

The notion of intuitionistic fuzzy set defined by K.T. Atanassov has been applied by Çoker (1997) for study intuitionistic fuzzy topological spaces. This concept has been developed by many authors (Çoker and co-workers, Lupiáñez,...).In various recent papers, F. Smarandache generalizes intuitionistic fuzzy sets and other kinds of sets to neutrosophic sets.

F. Smarandache also defined the notion of neutrosophic topology on the non-standard interval .One can expect some relation between the intuitionistic fuzzy topology on an IFS and the neutrosophic topology. We showed that this is false. Indeed, an intuitionistic fuzzy topology is not necessarilly a neutrosophic topology.

Also, Wang, Smarandache, Zhang, and Sunderraman introduced the notion of interval neutrosophic set, which is an instance of neutrosophic set and studied various properties. We study in this paper relations between interval neutrosophic sets and topology.

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Algebra and Algebraic Methods
FRIDAY 3, 16:00-19:30. Poster 8.

The exponential map of an algebraic vector field

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Abstract

In this work, we give a geometrical construction of the exponential map associated to an algebraic vector field defined on a k-scheme, valid in arbitrary characteristic. For this purpose, we make use of the functorial Cartier duality and the theory of finite inductive groups developed in [1] and [2].

Given a vector field on a k-scheme X, we construct an action of the (Cartier) dual of the additive group, G_a^*. In characteristic zero, this construction yields the classical action of the formal additive group, whether in the case of characteristic $p > 0$, these methods produce an action of G_a^* (which, as a scheme, is isomorphic to the countable sum $\bigoplus \alpha_p$, where $\alpha_p = k[x]/(x^p)$ stands for the additive group), satisfying analogous properties to the action the formal additive group in characteristic zero.

When the vector field is defined on a complete algebraic variety X, the image of the exponential map is an algebraic subgroup of the group of automorphism of X, closely related to the structure of the tangent algebraic subvarieties of the vector field.

We also compute this group in the case of the projective space, thus recovering, in the case of characteristic zero, the main results of [3], where this group was first introduced.

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Algebra and Algebraic Methods
FRIDAY 3, 16:00-19:30. Poster 9.

Indispensable binomials in toric ideals

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Abstract

Let $A \in \mathbb{N}^{d\times r}$ be a rank d matrix with non-negatives entries such that the row vector $(1,\dots,1)$ is a linear combination of the rows of A and let $B \in \mathbb{Z}^{r\times n}$ be an integer matrix such that the sequence $0 \to \mathbb{Z}^n \xrightarrow{B} \mathbb{Z}^r \xrightarrow{A} \mathbb{Z}^d \to 0$ is exact. Let $\mathcal{A}$ be submonoid of $\mathbb{N}^d$ generated by the columns $\mathbf{a}_1,\dots,\mathbf{a}_r$ of A.

Let $\Bbbk$ be any field. It is well known that the following set of binomials of $R := \Bbbk[x_1,\dots,x_r]$

$$\left\{\mathbf{x}^{\mathbf{u}} - \mathbf{x}^{\mathbf{v}} \mid \sum_{i=1}^{r} u_i\mathbf{a}_i = \sum_{i=1}^{r} v_i\mathbf{a}_i,\ \mathbf{u},\mathbf{v} \in \mathbb{N}^r\right\} \tag{1}$$

generates a homogenous toric ideal $I_{\mathcal{A}} \subset R$ which nothing but the kernel of the ring map $R \to \Bbbk[\mathbf{t}^{\mathbf{a}_1},\dots,\mathbf{t}^{\mathbf{a}_r}]; x_i \mapsto \mathbf{t}^{\mathbf{a}_i}$.

In Statistics, any (minimal) system of binomial generators of $I_{\mathcal{A}}$ can be regarded as a (minimal) Markov basis for certain connected Markov chains defined by A (see ([1])). In Integer Programming, some finite subsets of (1) determine Scarf test sets ([3]) for the following class of problems

$$\begin{gathered}\text{minize } \mathbf{b}_1\mathbf{z}\\ \text{subject to } \mathbf{b}_i\mathbf{z} \le c_i,\ i = 2,\dots,r,\end{gathered}$$

where $\mathbf{b}_i$ are the i-th row of B, $\mathbf{z} \in \mathbb{Z}^n$ and $\mathbf{c} = (c_1,\dots,c_r)' \in \mathbb{Z}^r$. Another subset of (1) of interest is the set of indispensable binomials of $I_{\mathcal{A}}$. A binomial $\mathbf{x}^{\mathbf{u}} - \mathbf{x}^{\mathbf{v}}$ is said to be indispensable if appears in any minimal system of binomial generators of $I_{\mathcal{A}}$ (notice that the set of indispensable binomials could be empty).

Using the results in [2], we investigate the existence of indispensable binomials and explore the relationship between the above subsets of (1). Furthermore, we give a necessary and sufficient condition for a toric ideal $I_{\mathcal{A}}$ to have a unique (up to scalars) Markov basis, that is to say, to be generated by its indispensable binomials. In fact, we prove that uniqueness of Markov bases implies uniqueness of minimal Scarf test sets, and vice versa.

*During the preparation of this work, both authors were partially supported by Ministerio de Educación y Ciencia (Spain), project MTM2007-65638.

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Statistics and Biometry
FRIDAY 3, 16:00-19:30. Poster 10.

A bivariate replacement policy for systems with a limited number of repairs

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Abstract

Consider a system subject to repairable failures, whose repair is perfect. Besides, we make a preventive maintenance by replacement at times nT, $n = 1, 2, \ldots$, where $T > 0$. For every replacement cycle of length T, only a maximum number $K - 1$, $K = 1, 2, \ldots$, of repairs can be made. If there are K failures in a replacement cycle, the system stops operating until next replacement. Costs are associated with the number of repairs per cycle, with the system replacement and with the system downtime. Reward is associated with the operating times. We assume that T is not a fixed quantity, but a variable one. The problem is to determine the optimal length of the replacement cycle and the optimal number of repairs allowed that maximizes the expected cost rate of the system.

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Functional Analysis
SATURDAY 4, 10:00-13:30. Poster 1.

Conorm and essential conorm in C*-algebras

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Abstract

Let A be a unital (complex) Banach algebra. An element a in A is called (*von Neumann*) *regular* if it has a generalized inverse, that is, if there exists b in A such that $a = aba$ and $b = bab$. In [2], the *conorm* of an element $a \in A$, denoted by $\gamma(a)$, is defined as the reduced minimum modulus of the left multiplication operator by a:

$$\gamma(a) = \gamma(L_a) = \inf\{\|ax\| \colon \operatorname{dist}(x, \ker(L_a)) = 1\}.$$

Regular elements in unital C*-algebras can be characterized as those elements having positive conorm ([1, 2]).

In [4], the *essential conorm* of a bounded linear operator T, on a complex Hilbert space H, $\gamma_e(T)$, is defined as the conorm of the element $\pi(T)$ in Calkin algebra $\mathcal{L}(H)/\mathcal{K}(H)$. One of the main results of [4] states that

$$\gamma_e(T) = \max\{\gamma(T + K) \colon K \in \mathcal{K}(H)\}.$$

We show how to generalize this result to the more general setting of unital C*-algebras.

Let A be a unital C*-algebra with non zero socle, which we denote by $\operatorname{soc}(A)$. We introduce the essential conorm of an element a in A (denoted by $\gamma_e(a)$) as the conorm of the element $\pi(a)$, where π denotes the canonical projection of A onto $A/\overline{\operatorname{soc}(A)}$. We prove that for every von Neumann regular element $a \in A$,

$$\gamma_e(a) = \max\left\{\gamma(a + k) : k \in \overline{\operatorname{soc}(A)}\right\}.$$

It is known that the conorm is upper semi-continuous in every unital C*-algebra, and that the only non-trivial continuity points of the conorm (respectively essential conorm) for the algebra of bounded linear operators on a Hilbert space are the one-sided invertible operators (respectively semi-Fredholm operators) (see [2, 4]). We characterize the continuity points of the conorm and essential conorm for extremally rich C*-algebras. We also obtain some formulae for the distance from zero to the generalized spectrum and Atkinson spectrum. In particular, this allows us to extend the main results in [4] and [3] to the wider scope of unital C*-algebras.

*This is a joint work with A. Peralta

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Algebra and Algebraic Methods
SATURDAY 4, 10:00-13:30. Poster 2.

An Observation on the Sum of Squares Question *

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Abstract

In 1998 Victoria Powers and T. Wörmann devised an algorithm to decide whether a given polynomial $p \in \mathbb{R}[x_1, ..., x_n] = R$ is a sum of squares in R. Later Pablo Parillo implemented similar ideas in his well known software package SOSTOOLS. This package makes efficient use of semidefinite programming. Indeed SOSTOOLS gives in case of $p \in \sum R^2$, a positive semidefinite real matrix S such that $p = z^T Sz$, for a certain vector of monomials in R. Using that $S = B^T B$ for some real positive semidefinite matrix one gets that the sum of squares of the vector Bz yields p. As an addendum to work of Mehrmann [M2] in this work it is shown that Gauss elimination without pivoting is possible for positive semidefinite matrices.While we do not claim the method as numerically the most advisable, it allows to obtain sum of squares representations in a more direct way, with more theoretical insight and much simpler than by the usual text book proposals. The result extends a theorem attributed for definite quadratic forms to Lagrange and Beltrami and is useful as a finishing step in recent algorithms by Powers and Wörmann [PW] and Parillo [PSPP] to write polynomials $p \in R$ as a sum of squares in R when such a representation exists.

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Statistics and Biometry
SATURDAY 4, 10:00–13:30. Poster 3.

Filtering and fixed-point smoothing algorithms for systems with multiplicative and additive noises*

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Abstract

Systems with multiplicative noise in the observations receive much attention in the literature since allow to model many situations associated with different physic phenomena such as image processing and communication systems between others.

The estimation problem based on observations with multiplicative noise has been widely treated by means of different approaches. Specifically, using a state-space approach, Nahi [1] derived an optimal linear recursive mean square error estimator for discrete-time systems with uncertain observations and Rajasekaran et al. [2] developed filtering and smoothing algorithms considering a white multiplicative noise for discrete and continuous-time systems. Recently, for discrete time-varying uncertain systems, Yang et al. [3] designed a robust finite-horizon Kalman filter assuming that the multiplicative and additive noises are white.

The above works assume that the multiplicative noise is white; however, there exist many practical situations, as for example in coherent images as ultrasound and synthetic aperture radar, where the multiplicative component is non white. Under the hypothesis of non-white multiplicative noise, the estimation problem has been also tackled by some researches; so Chow and Birkemeier [4] developed a linear recursive estimator based on the recursive form of the innovation process and recently, a new recursive filtering algorithm has been obtained by Zhang and Zhang [5] assuming that the additive noises of the state and measurement are correlated.

In many practical situations the equation generating the state process is unknown and it is not possible to use state-space approach to treat the estimation problem. In this

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case, the linear estimation problem can be solved using the information provided by the covariance functions of the processes involved as in Nakamori et al. [6] who, under the topic of uncertain observations, obtained recursive algorithms for estimating discrete-time signals.

In this paper, we tackle the estimation problem of a discrete signal, $u(k,l)$, from the observation model given by

$$z(k,l) = u(k,l) + u(k,l)w_1(k,l) + w_2(k,l), \quad \forall k,\ l \geq 1$$

where $z(k,l)$ is the observed signal in time k and localization l, $u(k,l)$ is the ideal signal and $w_1(k,l)$ and $w_2(k,l)$ are the multiplicative and additive noises, respectively. In order to derive the linear estimators of the signal, we assume that the multiplicative noise is a colored process since this assumption, as we have already commented, is real to model some image formations. Moreover, we consider that the signal and additive noise are correlated; this hypothesis, as Budhiraja [7] comment, is valid to model many practical situations in engineering and image processing. Under these hypotheses on the noises and assuming the equation which describes the signal is unavailable, we propose recursive filtering and fixed-point smoothing algorithms based on the information provided by the covariance function of the processes involved.

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Algebra and Algebraic Methods
SATURDAY 4, 10:00-13:30. Poster 4.

Some additive results on Drazin Inverses

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Abstract

Our aim is to investigate the existence of the Drazin inverse $(p+q)^d$ of the sum $p+q$, where p and q are either ring elements or matrices, and a^d denotes de Drazin inverse of a. We recall that a^d is the unique solution, if it exists, to $a^k xa = a^k, xax = x, ax = xa$, for some integer $k \geq 0$. We will give sufficient conditions in order to $p + q$ be Drazin invertible, generalizing recent results ([3], [6]), and give converse results assuming the ring is Dedekind-finite.

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Statistics and Biometry
SATURDAY 4, 10:00-13:30. Poster 5.

Nonlinear estimation in systems with uncertain observations with correlation in the uncertainty*

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Abstract

The estimation problem in nonlinear dynamic systems constitutes an interesting and growing research area due to the numerous physical systems such as guidance or navigational systems, target tracking or even growth models, that present some kind of nonlinearity in the equations describing them. Due to the difficulty to compute the optimal solution, many approximations have been developed and applied to different practical problems; such approximations include the extended Kalman filter, the unscented Kalman filter, Gaussian sum filters and particle filters. A survey comparing these and other nonlinear filters is described in [1].

The most commonly used approximation to the estimation problem in nonlinear systems is the extended Kalman filter (EKF), which linearizes the nonlinear functions appearing in the description of the system using their Taylor series expansions; therefore, if the model is highly nonlinear, the EKF can introduce large errors in the estimated statistics of the posterior distributions of the states. Besides others, the unscented Kalman filter (UKF) [2] becomes solvent the deficiencies of linearization of the EKF by providing an algorithm based on a direct and explicit mechanism for transforming the mean and covariance information when a nonlinear function is considered; many practical applications have shown the performance of the unscented filter to be clearly superior to that of the extended filter.

In standard filtering (linear and nonlinear) problems it is usually assumed that the signal to be estimated is always present in the observations. However, it frequently occurs that the measurement mechanism, or the transmission of such measurements, is subject to random failures, generating observations which consist in noise only. Since it is not generally known whether the observation used for estimation contains the signal or it consists only of noise, the observation equation is designed by including a binary random multiplicative noise which models this uncertainty.

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Although the estimation problem from uncertain observations has been extensively studied in linear systems (see [3, 4] and references therein), the literature on the nonlinear filtering with uncertainty is fairly limited, with the exception of a few results such as those reported in [5] and [6]. This paper examines an extension of the nonlinear filtering problem addressed in [6] for the case in which the uncertainty in the observation process is modelled by Bernoulli random variables which are correlated at consecutive time instants being independent in other case. An extension of the unscented Kalman filter is proposed as an approximation to the optimal filter; successive applications of the unscented transformation to a suitable augmented state vector permit to approximate the one-stage state and observation predictors from the state filter at the previous time instant.

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Statistics and Biometry
SATURDAY 4, 10:00–13:30. Poster 6.

Signal smoothing via mixture approximations from uncertain observations with signal-noise correlation*

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Abstract

Systems with uncertain observations are characterized by the fact that the signal is not always present in the observations, but there exists a non-zero probability that the signal is missing. This kind of systems are appropriate to model a large class of real situations with intermittent failures in the measure mechanism, accidental loss of some observations, or inaccessibility of the data during certain time periods. They are modeled by including in the observation equation, besides the usual additive noise, a multiplicative noise component described by a sequence of Bernoulli random variables whose values -one or zero- indicate the presence or absence of the signal in the observation.

Usually, in this kind of systems it is assumed that the signal process and the observation noise are independent; nevertheless, this assumption is not realistic in a large class of real-world situations concerning models with correlated signal and noise (see e.g. [1]-[3]). The least-squares (LS) optimal signal estimation problem in these systems is focused in the search of suboptimal estimators; this is due to the fact that the optimal estimator, which is the conditional expectation of the signal given the observations, is not easily achievable since the joint distribution of the signal and the observations is not Gaussian (even when the signal and the additive noise are both Gaussian processes). For example, in [4] and [5], the LS linear estimation problem is solved assuming that the state-space model is not completely known, but only using covariance information, and that the signal and the observation additive noise are correlated. Under these same hypotheses, recently, Nakamori et al. [6] have proposed a suboptimal nonlinear filtering algorithm to estimate Gaussian

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signals; this algorithm is derived approximating the conditional distribution of the signal given the observations via successive approximations of mixtures of Gaussian distributions.

In this paper, we approach the fixed-point and fixed-interval smoothing problems of Gaussian signals assuming the same hypotheses as that considered in [6] and using the same methodology.

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Functional Analysis
SATURDAY 4, 10:00-13:30. Poster 7.

Invertibility for a class of convolution type operators and solutions for associated corona problems with n-AP data

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Joint with M.A. Bastos

Abstract

The invertibility for classes of convolution type operators is established [1] by means of their relations to Toeplitz operators and the solution of associated corona problems. The explicit inverse of the Toeplitz operators is obtained, for a class, through factorization of the corresponding $n \times n$ matrix function, which, in turn is written by solving the corresponding n dimensional Riemann-Hilbert problem. Using the relations between the existence of factorization for matrices with constant determinant and the existence of solution for certain corona problems, a class of corona problems with n unknowns is solved where the data are $n-1$ $AP-$monomials and one AP polynomial. The explicit solution for those problems is written using the previous factorization.

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Statistics and Biometry
SATURDAY 4, 10:00–13:30. Poster 8.

The problem of matching two configurations by using a Bayesian non-linear model

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Abstract

In Shape and Procrustes Analysis a typical problem is how to match two or more configurations of labelled points or landmarks by applying a geometrical transformation. We present the problem of matching two configurations by using a neural net, under normality, from a Bayesian point of view and we derive the posterior distributions of the parameters. This model generalizes, in terms of a neural network, previous linear matching models.

As a practical application, we analyze data of patients with Acute Lymphoblastic Leukemia from database ALL. We find out the best neural net model that relates expression levels of two types of cytogenetically different samples.

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Functional Analysis
SATURDAY 4, 10:00-13:30. Poster 9.

Fredholm theory for periodic diffraction boundary-value problems of first order

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Joint with M.A. Bastos

May 2008

Abstract

Boundary-value problems of first order for the Helmholtz equation are considered within the context of wave diffraction by a periodic strip grating and formulated as convolution type operators acting on a Bessel potential periodic space setting. Three boundary-value problems are studied for an arbitrary geometry of the grating: the oblique derivative, the mixed oblique derivative/Neumann and the classic Neumann boundary-value problems. The convolution type operators on the grating which correspond to the given boundary-transmission problems are associated with Toeplitz operators acting on spaces of matrix functions defined on composed contours. For these periodic boundary-value problems a Fredholm theory is developed and the Fredholm indices are deduced.

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Algebra and Algebraic Methods
SATURDAY 4, 10:00-13:30. Poster 10.

Clifford codes and fully ramified characters

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Abstract

Stabilizer codes are the most common construction of quantum error correcting codes. Clifford codes can be defined as an homogeneous component of the abstract error group G. They are a generalization of stabilizer codes, in fact, if Q is a Clifford code and the quotient group $G/Z(G)$ is abelian, then Q is a stabilizer code.

In this work we extend the characterization of those Clifford codes which are stabilizer codes given in [8]. We prove that this property is equivalent to the existence of a fully ramified character χ of a normal subgroup N of G. We also define a new kind of non-stabilizer Clifford codes, called product codes, using methods of classic theory of codes and we study their correction properties applying group theoretical techniques.

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Statistics and Biometry
SATURDAY 4, 10:00–13:30. Poster 11.

Bayesian analysis for exponential families by using mixtures of prior distributions

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Abstract

The choice of suitable prior distributions is often a contentious task in the situations where Bayesian methods are applied. Very often the prior distribution is chosen to approximately reflect the initial expert's opinion. In this context, a common choice is a conjugate prior distribution. However, when this choice is mainly based on "mathematical convenience", then the conjugate family might not be adequate enough to model a certain kind of a prior beliefs (see [2]).

[4] and [5] showed that it is possible to extend this family through the use of mixtures of conjugate prior distributions. There are two main advantages. On one side, mixtures of conjugate distributions are also conjugate families, and therefore mixtures of conjugate prior distributions can be treated as regular conjugate distributions. On the other side, mixtures of conjugate prior distributions lead to greater freedom and flexibility when modeling the prior information. Some interesting applications, based on mixtures of conjugate prior distributions, can be found in [6].

In this work, a general approach to address a Bayesian analysis by using mixtures of conjugate prior distributions is proposed. Natural exponential families with quadratic variance function (NEF-QVF) are used because they include distributions of wide use in real applications, see, for example, [1].

It is assumed that the prior information comes from several sources such as experts. The opinion of each expert is elicited as a conjugate distribution over a quantity of interest. Then, the opinions are combined by using a mixture of distributions. [3] observed that the weights selection is an inconvenient of this approach. In some papers, the weights are fixed in advance. Here, unknown weights are considered and a general procedure based on Kullback-Leibler (K-L) distances to obtain them is proposed. The main advantage is that the process is analytical.

Finally, the discrepancies between the combined posterior belief and each expert's prior belief are analyzed by using K-L distances between the mixture of the posterior distributions and the prior distribution for each expert. A Monte Carlo-based approach is considered to estimate the K-L distances. The estimate can be split in two terms one that has to be approximated and another one that is exact. Note that, the non-analytical term is the same for all experts.

Illustrative examples are presented to show that the proposed techniques are easily applied in practice.

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Statistics and Biometry
SATURDAY 4, 10:00–13:30. Poster 12.

Matrix multiplication using group theoretical techniques

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Abstract

Group-theoretical algorithms for multiplying matrices are similar to fast algorithms for multiplying polynomials. Both consist on an embedding into a group algebra $\mathbf{C}[G]$ where the product is performed. For polynomials, the product consists in pointwise multiplication whereas for matrices is a block-diagonal matrix multiplication.

The group G determines the complexity of the algorithm. For instance, the product of square matrices of order n can always be implemented using an abelian group of order n^3, but the bound obtained for complexity is trivial, i. e., $O(n^3)$. Using non-abelian groups it is possible to achieve better bounds for the complexity, that is, $O(n^\omega)$ with $\omega < 3$.

In this work we review the methods in [1] and [2]. We also discuss the relations between the combinatorial properties of the group G and the complexity of the algorithms for matrix multiplication. Finally, we construct an algorithm based on the ideas exposed in [2].

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Algebra and Algebraic Methods
SATURDAY 4, 16.00-19.30. Poster 1.

On the simplicity and some identities of a ternary quaternion algebra and its reduced algebras

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Abstract

A few years ago, the theory of $n-$Lie algebras attracted a lot of attention because of its close connection with the Nambu Mechanics, [2]. This connection was revealed in [4], where these algebras were called Nambu-Lie algebras. The notion of $n-$Lie algebra ($n \geq 2$) was introduced, in 1985, by the russian mathematician Valerii Terent'evich Filippov, [1]. He proposed a natural generalization of the Lie algebra concept; in fact, both notions agree for $n = 2$. Following [3], we use the term Filippov algebra instead of $n-$Lie algebra.

In this work, we consider the ternary Filippov algebra A_1 (vector cross product algebra) equipped with a bilinear, symmetric and non-degenerate form and the orthonormal canonical basis. We define a new multiplication on the underlying vector space of A_1. This new algebra, A, appears analogously to the construction of the quaternions from the Lie algebra sl_2. Moreover, it's an enveloping algebra for the Filippov algebra A_1, [3]. We prove the simplicity of A and then deduce its identities of degree 1 (including the minimal one) and some of degree 2. Further, we establish a few results concerning the identities of degrees 1 and 2 of the reduced algebras of A.

References

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Algebra and Algebraic Methods
SATURDAY 4, 16.00-19.30. Poster 2.

Homology of Leibniz n-algebras

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Abstract

Leibniz n-algebras were introduced in [3] as a non-skew-symmetric version of Nambu algebras, which naturally arose in the so called Nambu mechanics [4]. They are K-vector spaces $\mathcal{L}$ equipped with a n-linear bracket $[-,\dots,-]:\mathcal{L}^{\otimes n}\to\mathcal{L}$ satisfying the following fundamental identity

$$[[x_1,x_2,\dots,x_n],y_1,y_2,\dots,y_{n-1}]=\sum_{i=1}^{n}[x_1,\dots,x_{i-1},[x_i,y_1,y_2,\dots,y_{n-1}],x_{i+1},\dots,x_n] \quad (1)$$

In case $n=2$ the identity (1) is the Leibniz identity, so a Leibniz 2-algebra is a Leibniz algebra.

In the present work, we introduce the notion of co-representation for Leibniz n-algebras which is equivalent to a left module over the universal enveloping algebra introduced in [2]. This notion in case $n=2$ gives the corresponding notion of co-representation of Leibniz algebras in [5]. Then we construct a complex of a Leibniz n-algebra $\mathcal{L}$ over a co-representation M by means of the Leibniz complex of $\mathcal{L}^{\otimes n-1}$ over the co-representation M$\otimes\mathcal{L}$. In case $n=2$ we obtain the Leibniz homology developed in [5]. When M is the trivial co-representation K, then we obtain the homology with trivial coefficients in [1].

We prove the vanishing of the homology over free objects and a result which is a generalization to Leibniz n-algebras of the following isomorphism $HL_\star(\mathcal{L},\mathcal{L})\cong HL_{\star+1}(\mathcal{L},K)$ for Leibniz algebras.

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Algebra and Algebraic Methods
SATURDAY 4, 16.00-19.30. Poster 3.

Solvable and Nilpotent Algebras With Bracket

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Abstract

Algebras with bracket are associative (non necessarily commutative) algebras equipped with a bilinear operation $[-,-]$ satisfying the (left) Poisson distribution law: $[a \cdot b, c] = [a, c] \cdot b + a \cdot [b, c]$. Poisson algebras are examples of algebras with bracket. It was proven in [2] that the operad corresponding to algebras with bracket is Koszul. A homology with trivial coefficients and its application to study universal central extensions of algebras with bracket was the subject of [1].

In the present work we introduce several necessary material on algebras with bracket and we continue with the definition, properties and examples of solvable and nilpotent algebras with bracket. After this, we apply the homological machinery developed in [1] in order to obtain a characterization of nilpotent algebras with bracket in an analogous way to Stalling's theorem for Lie algebras [3].

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Statistics and Biometry
SATURDAY 4, 16.00-19.30. Poster 4.

On Nordlander's conjecture in the three-dimensional case

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Abstract

Let X be a real normed space and let $S = \{x : x \in X, ||x|| = 1\}$. The modulus of convexity of X is the function $\delta_X : [0,2] \to [0,1]$, defined by

$$\delta_X(a) = \inf\{1 - \frac{1}{2}||x+y|| : x, y \in S, ||x-y|| = a\}.$$

It follows from Jordan-von-Neumann parallelogram law, that if H is an inner product space then $\delta_H(a) = 1 - \frac{\sqrt{4-a^2}}{2}$. Nordlander discovered [1], that for an arbitrary normed space X the following inequality is true

$$\delta_X(a) \leq \delta_H(a), \forall a \in (0,2).$$

This inequality is called Nordlander's inequality and shows that i.p.s. are "the most uniformly convex" spaces in the class of normed spaces [4].

In Day [3, Th. 4.1] it is proved that if

$$\delta_X(a) = \delta_H(a), \forall a \in (0,2)$$

then X is an i.p.s.. Nordlander conjectured [1], that the same conclusion should hold if the above equality takes place for some fixed $a \in (0,2)$. Alonso and Benitez [2], making use of real two-dimensional arguments, proved the validity of Nordlander's conjecture for $a \in (0,2) \setminus D$, where $D = \{2\cos(k\pi/(2n)) : k = 1, \ldots, n-1; n = 2, 3, \ldots\}$, and they gave counterexamples for $dimX = 2$ and $a \in D$. However, according to [5, p.154], the validity of Nordlander's conjecture for any $a \in (0,2)$ is still open in the case $dimX \geq 3$ and in the present report we give an affirmative answer on it.

Theorem. *Let X be a real normed space with $dimX \geq 3$ and let a be a fixed number from the interval $(0,2)$. Then X is an inner-product spaces if and only if $\delta_X(a) \geq 1 - \frac{\sqrt{4-a^2}}{2}$.*

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Statistics and Biometry
SATURDAY 4, 16.00-19.30. Poster 5.

On the quasi-greedy orthonormal basis in $L^{p(t)}([0,1])$ Spaces

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Abstract

Let $(x_n)_{n\in\mathbb{N}}$ be a semi-normalized basis in a Banach space X. This means that (x_n is a Schauder basis and is semi-normalized i.e. $0 < \inf_{n\in\mathbb{N}} \|x_n\| \leq \sup_{n\in\mathbb{N}} \|x_n\| < \infty$. For an element $x \in X$ we define the error of the best $m-$term approximation as follows

$$\sigma_m(x) = \inf\{\|x - \sum_{n\in A} \alpha_n x_n\|\},$$

where the inf is taken over all subset $A \subset \mathbb{N}$ of cardinality at most m and all possible scalars α_n. The main question in approximation theory concerns the construction of efficient algorithms for m-term approximation. A computationally efficient method to produce m-term approximations, which has been widely investigated in recent years, is the so called greedy algorithm. We define the greedy approximation of $x = \sum_n a_n x_n \in X$ as

$$\mathcal{G}_m(x) = \sum_{n\in A} a_n x_n,$$

where $A \subset \mathbb{N}$ is any set of the cardinality m chosen in such a way that $|a_n| \geq |a_l|$ whenever $n \in A$ and $l \overline{\in} A$. We say that a semi-normalized basis $(x_n)_{n\in\mathbb{N}}$ is greedy if there exists a constant C such that for all $m = 1, 2, ...$ and all $x \in X$ we have

$$\|x - \mathcal{G}_m(x)\| \leq C\sigma_m(x).$$

This notion evolved in theory of non-linear approximation see e.g.[1],[2]. A result of Konyagin and Temlyakov [3] characterizes greedy bases in a Banach spaces X as those which are unconditional and democratic, the latter meaning that for some constant $C > 0$

$$\left\|\sum_{\alpha\in A} \frac{x_\alpha}{\|x_\alpha\|}\right\| \leq C\left\|\sum_{\alpha\in A'} \frac{x_\alpha}{\|x_\alpha\|}\right\|$$

holds for all finite sets of indices A, $A' \subset \mathbb{N}$ with the same cardinality.

Wavelet systems are well known examples of greedy bases for many function and distribution spaces. Indeed, Temlyakov showed in [1] that the Haar system (and any wavelet system L^p-equivalent to it) is greedy in the Lebesgye spaces $L^p(\mathbb{R}^n)$ for $1 < p < \infty$. When wavelets have sufficient smoothness and decay, they are also greedy bases for the more general Sobolev and Tribel-Lizorkin classes (see e.g.[4-5]).

A bounded Schauder basis for a Banach space X is called quasi-greedy if there exists a constant C such that for $x \in X$, $\|\mathcal{G}_m(x)\| \leq C\|x\|$ for $m \geq 1$

The purpose of this report is to construct the quasi-greedy basis in the class of variable exponent Lebesgue spaces $L^{p(t)}([0,1])$.

we define $p^+ = esssup_{x\in[0,1]}p(x)$ and $p^- = essinf_{x\in[0,1]}p(x)$.

We prove following

Theorem 1. Let $p(\cdot) : [0,1] \longrightarrow [1,\infty)$ be a measurable function such that $1 < p^- \leq p^+ \leq 2$and Hardy-Littlewood maximal operator is bounded on the space $L^{p(t)}([0,1])$. Then there exists a uniformly bounded orthonormal quasi-greedy basis in $L^{p(t)}([0,1])$.

Theorem 2. Let $p(\cdot) : [0,1] \longrightarrow [1,\infty)$ be a measurable function such that $2 \leq p^- \leq p^+ < \infty$ and Hardy-Littlewood maximal operator is bounded on the space $L^{p(t)}([0,1])$. Then there exists a uniformly bounded orthonormal quasi-greedy basis in $L^{p(t)}([0,1])$.

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Statistics and Biometry
SATURDAY 4, 16.00–19.30. Poster 6.

On the L^2-convergence in a class of homogeneous multitype Markov chains

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Abstract

The class of homogeneous multitype Markov chains with states having non-negative integer coordinates (HMMC) has been recently considered by the present authors in [1]. As they point out in that work, this class, or the more general of Markov population processes, is wide enough to include most of the branching processes. Moreover, the theory underlying these branching processes can be generalized and abstracted in such a way that the main results of such theory are mere consequences of their analogous for HMMC. This fact is shown in [1], where the study of the extinction for controlled multitype branching processes becomes a corollary of the investigations on the unlimited growth of HMMC.

Furthermore, in population dynamics, we can find situations where it is not realistic to assume an observable branching scheme. This is the case of some bacterium colonies where it is possible to count the number of individuals in each generation but not to monitor the different family lines. Also, in most of the cases, reproduction of each individual can not be considered independent of the rest, for instance, when there exist overpopulation problems.

Another systems do not even fit a branching scheme, like for example the evolution of certain diseases in which individuals could be infected by successive contacts with several sick individuals, and none of them is the only responsible of the illness of the first one.

These situations, whose modelling is complicated through branching processes, fit perfectly to a HMMC, since its only assumption is the Markov property, i.e. what happens in a time $n+1$ depends only on the state of the system at time n, without further specifications about the nature of this dependence.

Bearing this fact in mind, the present authors have made contributions to the investigation of the extinction/explosion (see [1]) and the different kinds of asymptotic behaviour of HMMC (see [2] and [3]). More specifically, in [2] the geometric growth of HMMC has been researched. In fact, considering a situation that could be called supercritical, these authors have proved the almost sure and L^α-convergence, for $1 \leq \alpha \leq 2$, of the suitably

normed chain to a non-null random vector. In order to obtain those results, the main tools are Cauchy-Schwarz type inequalities.

It seems natural to find out other specific conditions for the L^2-convergence, taking advantage of the Hilbertian properties of the L^2-space. The present communication deals with this problem, providing both sufficient and necessary conditions for the almost sure and L^2-convergence of a suitably normed HMMC to a non-degenerate limit. These conditions will depend on the transition mean vectors and the transition covariance matrices of the chain. For some cases, we shall show that the conditions imposed are an improvement on those given in [2].

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Statistics and Biometry
SATURDAY 4, 16.00-19.30. Poster 7.

A Bertalanffy diffusion model*

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Abstract

Growth is an important characteristic in many application fields. The study of this phenomenon was originally associated with the evolution of animal populations. Nowadays, however, it is considered in many contexts, for instance, in those of economics, biology and ecology. For this reason, many attempts have been made to build mathematical models to describe this kind of behavior.

Von Bertalanffy [5] introduced a growth equation to model fish weight growth, from an adaptation of the Verhulst logistic growth, by assuming a maximal weight which might eventually attained, being the growth rate proportional to the difference between maximal and current weight. Actually, it is the most common model used by fishery biologists to study growth in fishes and its interpretations, such as fish population dynamics, and the effects of fishery regulations on the catch. For example, the length of fishes can be studied in a similar form taking into account the existing relation between the weight and the length of fishes.

The most general expression for the Bertalanffy curve is

$$f(t) = f_\infty \left[1 - e^{-k(t-t_0)}\right]^b; \quad t \geq t_0;\, k, b > 0;\, t \geq t_0 \geq 0$$

where f_∞ is the upper bound for the variable under study and that can only be reached after infinity time. The case $b = 1$ is used in order to study length of fishes, whereas the case $b = 3$ is employed for weight.

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Because of the importance of this curve, the study of procedures in order to determine its parameters has been widely considered. For example, Rafail, [4], developed a procedure based on a straight-line relationship between the natural logarithms of growth increments per unit of age against age as the independent variable. Nevertheless, these type of procedures do not take into account the fluctuations or disturbances that might exist in the system under consideration. These disturbances can come from multiple factors, which are not always quantifiable or may even be unknown. As a first approach in this line, some authors, as Kimura [2], have studied likelihood methods under the assumption of independent and normally distributed errors using classic nonlinear least squares methods.

In recent decades the aim of model formulation has been focussed on modeling both the trend of growth processes and random environmental effects, by means of stochastic differential equations. These equations include a noise term in the ordinary differential equation associated to the deterministic model, and their solutions are diffusion processes. In this context, and as the Von Bertalanffy growth model concerns, recently Qiming and Picthford [3] have proposed some stochastic models built from stochastic differential equations with identical von Bertalanffy deterministic parts and different stochastic terms in accordance with different behaviors of the fish size. However, they do not consider procedures for fitting empirical data to the models.

In this paper we propose a new Bertalanffy-type diffusion process by applying the methodology developed by Gutiérrez et al. [1] in the context of the gompertzian growing. The case of non degenerate initial distribution will deserve a special attention, because that will allow us to describe and to model phenomena associated with time-continuous variables showing a Bertalanffy-type behaviour and with the additional characteristic of having the upper bound depending on the initial value. An inferential study of the parameters of the process is carried out on the basis of discrete sampling, including some strategies in order to solve the likelihood equations. Finally, some examples are also presented.

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Statistics and Biometry
SATURDAY 4, 16.00-19.30. Poster 8.

Stochastic square- Brennan- Schwartz diffusion process: Statistical inference

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Abstract

In this paper, on the basis of the Brennan-Schwartz model which is widely considered in stochastic finance (see, e.g, [1], [2] and [3]), we present a stochastic process that we call square- Brennan- Schwartz diffusion process, which is defined as a one-dimensional stochastic process $\{x(t); t \in [t_0, T]; t_0 \geq 0\}$ taking values on $(0, \infty)$ and governed by the following Ito's stochastic differential equation (SDE):

$$dx(t) = \left(\alpha x(t) + \beta\sqrt{x(t)}\right) dt + \sigma x(t) dw(t) \quad , \quad x(t_0) = x_{t_0}$$

where $\sigma > 0$, α and β are real parameters, $w(t)$ is a one-dimensional standard Wiener process and $x_{t_0} > 0$ is a fixed real.

Under known regularity conditions, the process $\{x(t); t \in [t_0, T]; t_0 \geq 0\}$ solution of the afore mentioned SDE is the homogenous diffusion process characterized by the infinitesimal moments drift and diffusion coefficient (volatility) given respectively by:

$$a(x) = \alpha x + \beta\sqrt{x} \quad \text{and} \quad b(x) = \sigma^2 x^2$$

This process is an extension of the homogenous lognormal diffusion process, that can be obtained when $\beta = 0$. On the other hand, it is proved by stochastic calculus that the proposed process can be generated by means of a type square transformation of the Brennan-Schwartz model. This justifies the adopted denomination for the process studied in this work.

Firstly, we determine the basic probabilistic characterizations of the process as its analytical expression and their trend functions (conditional and non conditional).

Secondly, we estimate the parameters present in the model, the drift parameters (α and β) are estimated by using the maximum likelihood estimation method in basis of continuous sampling of the process (see, e.g, [2], [4] and [6]) and the diffusion coefficient parameter can be estimated by a similar approximation procedure used in [5] and [7].

Later, we study the ergodicity problem of the process, showing, in particular, that the stationary distribution of the process is the inverse of the square root of a gamma distribution, and obtaining the asymptotic normality of the likelihood estimators for drift parameters and the approximated asymptotic confidence intervals for the drift parameters.

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Statistics and Biometry
SATURDAY 4, 16.00-19.30. Poster 9.

Determining the optimum calibration point in estimating the distribution function

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Abstract

In survey sampling a common problem is the estimation of the finite population distribution due such as the finite population quantiles and some poverty measures can be obtained by means of this function.

In the presence of auxiliary information, there exist several general estimation procedures in recent literature to obtain more efficient estimators for the population means and totals. Effort has been made to directly apply these general procedures for the estimation of the distribution function. However, due to specific nature of a distribution function, the resulting estimators often have some undesirable properties.

The design-based ratio $\widehat{F}_r(t)$ and difference $\widehat{F}_d(t)$ type estimators for the distribution function (Rao et al, 1990) suffer from several drawbacks, the obvious one being that they can take values outside $[0, 1]$. Furthermore, they are not always monotone functions and therefore are not recommended for estimating finite population quantiles. Other important papers on this topic (Chambers and Dunstan, 1986, Dorfman and Hall, 1993) have assumed a superpopulation model, and have suggested model-based estimators. Careful model checking and diagnostics need to be carried out before these model-based estimators are used.

In the last decade, calibration estimation (Deville and Särndal, 1992, Singh 2003,...) has developed into an important field of research in survey sampling. Calibration is now an important methodological instrument in the production of statistics. Some major advantages of this technique come given in the work of Särndal, 2007. As Särndal point out calibration has establisher itself as an important methodological instrument in large-scale production of statistics. Several national statistical agencies have developed software designed to compute the calibration weights.

Recently, Rueda et al. (2007) use the calibration technique proposed by Deville and Särndal (1992) to propose an estimator of the distribution function assuming a linear relationship of the variables under study. This estimator is built by means of constraints that require the use of a fixed value t_0. The accuracy of the calibration estimators depends on the t_0 value. In addition this estimator suffer a loss of efficiency when t_0 is far away from t, the point where the distribution function is being evaluated.

In this paper we analyze the problem of determining the optimum value where do the calibration, so that the estimator produce more accurate estimates. The study was conducted for a general sampling design. We propose an algorithm to determine, using the sample data, how to choose that point. The work ends with a practical application of method.

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Functional Analysis
SATURDAY 4, 16.00-19.30. Poster 10.

On the Distribution of zeros of Entire Exponential Type Almost-Periodic Functions

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Abstract

We present a result on entire exponential type almost-periodic functions that settles the exact number of their zeros in an arbitrary rectangle of the critical strip where are situated all the zeros with an error of ± 2.

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Statistics and Biometry
SATURDAY 4, 16.00–19.30. Poster 11.

Improving models for the prediction of liquid saturation densities

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Abstract

The calculation of saturated densities of pure substances at vapor–liquid equilibrium is essential for important practical applications, and serves as the basis for calculating other properties such as the surface tension.

The simplest empirical correlations are based on applying the corresponding states principle (CSP) with the critical density and temperature, and the acentric factor as input data.

The accuracy and applicability of these models to different kinds of fluids, in the temperature range where experimental data are available, have been extensively studied. Unfortunately, the obtained results show that there is no universal model that yields very accurate results for all kinds of fluids. Moreover, the models based on the CSP do not always give accurate results far from the critical point, in particular, near the triple point.

In this work we use simple modifications of several of those models in order obtain good accuracy at any temperature. In particular, for each model we change a fixed coefficient to a variable coefficient, which permits the model to reproduce exactly the value of the density at the triple point. The accuracy of a model based on a scale-variable-reduced-coordinate framework, and of a new simple predictive model recently proposed by us is also checked. This latter model does not contain adjustable coefficients but only two variable coefficients that depend on the temperature and density at both the critical and the triple point.

The values obtained by original and modified models were compared with the data accepted in the DIPPR database for 107 fluids. These fluids were selected by taking into account that an adequate number of accepted data are avaiable over a wide temperature range.

Our results indicate that the new models clearly improve the estimates in the whole temperature range with respect to the classical CSP ones and some others using the triple point as reference. Indeed, we showed that a simple modification of the well-known and straightforward Rackett model gives an excellent overall accuracy (mean average absolute deviation 0.9%) for the fluids considered.

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Algebra and Algebraic Methods
SUNDAY 5, 10.00-13.30. Poster 1.

Matrix representation of period doubling cascade

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Abstract

Period doubling cascade [1, 2] is one of the most common phenomena in dynamical systems and perhaps the most important mechanism of transition to chaos. That is why any mathematical reformulation of the process can help researchers reformulate old problems or approach new ones.

We have proved that the period doubling cascade admits a matrix representation. The block expression of the matrix corresponding to the 2^k-periodic orbit is a 2^k order matrix given by

$$A_{2^k} = \begin{pmatrix} & A_{2^{k-2}} & \\ & & I_{2^{k-2}} \\ I^*_{2^{k-1}} & & \end{pmatrix} \quad \forall\, k \geq 3$$

where $A_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $A_{2^2} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$, and $I_{2^{k-2}}$ is the 2^{k-2}-order identity matrix

and $I^*_{2^{k-1}} = \begin{pmatrix} & & 1 \\ & 1 & \\ \iddots & & \\ 1 & & \end{pmatrix}$ is a submatrix of order 2^{k-1}.

The recursive expression shown by the above matrices is due to the fact that matrices are obtained from permutations [3] satisfying the recursive expression

$$\begin{array}{c} \sigma_{2^k}(2n-1) = \sigma_{2^{k-1}}(n) \\ \sigma_{2^k}(2n) = 2^k + 1 - \sigma_{2^{k-1}}(n) \end{array} \quad 1 \leq n \leq 2^{k-1} \quad n, k \in \mathbb{N}$$

where $\begin{array}{l} \sigma_2(1) = 1 \\ \sigma_2(2) = 2 \end{array}$.

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Statistics and Biometry
SUNDAY 5, 10.00–13.30. Poster 2.

A data-based method for choosing the number of pilot stages for plug-in bandwidth selection

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Abstract

The choice of the bandwidth is a crucial issue for kernel density estimation. Among all the data-dependent methods for choosing the bandwidth, the plug-in method has shown a particularly good performance in practice. This procedure is based on estimating an asymptotic approximation of the optimal bandwidth, using two 'pilot' kernel estimation stages. Although two pilot stages seem to be enough for most densities, for a long time the problem of how to choose an appropriate number of stages has remained open. Here we propose an automatic (i.e., data-based) method for choosing the number of stages to be employed in the plug-in bandwidth selector.

Algebra and Algebraic Methods
SUNDAY 5, 10.00-13.30. Poster 3.

Structural patterns of autotopisms of maximum rank quasigroups

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Abstract

A *quasigroup* is a nonempty set Q endowed with a product $\cdot$, such that if any two of the three symbols a, b, c in the equation $a \cdot b = c$ are given as elements of Q, the third is uniquely determined as an element of Q. If there exists $e \in Q$ such that $a \cdot e = e \cdot a = a$, for all $a \in Q$, then Q is a *loop*. Johnson and Smith [2] extended the traditional character theory for finite groups to finite quasigroups. To do it, they defined the *conjugacy classes* of a quasigroup Q as the orbits of the diagonal action of the multiplication group on Q^2, of which are indeed a partition. Thus, the number of conjugacy classes is the *rank* of Q and it is verified that almost all finite quasigroups have rank 2 [3].

In this paper, we study some properties of the set $\mathcal{Q}(n)$ of those quasigroups of n elements having maximum rank n. Although one such a quasigroup Q must be a loop, the reciprocal is false in general. Thus, we can use the existence of an unit element of Q in order to study the symmetrical structure of its multiplication table, given by the autotopism group of Q, $\mathcal{A}(Q) = \{(\alpha, \beta, \gamma) \in S_n^3 \mid \alpha(a) \cdot \beta(b) = \gamma(a \cdot b),\ \forall a, b, c \in Q\}$, where S_n denotes the permutation group on the elements of Q. Specifically, the *cycle structure of a quasigroup autotopism* $(\alpha, \beta, \gamma) \in S_n^3$ is the triple $(\mathbf{l}_\alpha, \mathbf{l}_\beta, \mathbf{l}_\gamma)$, where, given $\delta \in \{\alpha, \beta, \gamma\}$, $\mathbf{l}_\delta = (l_1^\delta, l_2^\delta, ..., l_n^\delta)$ is the sequence of the number of cycles of length i in δ, for all $i \in \{1, 2, ..., n\}$. Some general properties and a classification of all cycle structures of autotopisms of quasigroups with at most 11 elements are given in [1]. In this paper, we complete the mentioned work by analyzing, in a first step, the cycle structures of the set of loop autotopisms. Then, we impose the condition of having maximum rank in order to obtain a classification of all possible structural patterns of our set $\mathcal{Q}(n)$ for $n \leq 11$. Finally, we give an outline about the application of all the previous results in the calculus of the character tables of the quasigroups of $\mathcal{Q}(n)$ and their corresponding determinant groups.

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Functional Analysis
SUNDAY 5, 10.00-13.30. Poster 4.

On an estimate for the dimension of the kernel of a singular integral operator with a non-Carleman shift

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Abstract

We will consider the operator $T = I - cUP_+$: $L_2^n(\mathbb{T}) \to L_2^n(\mathbb{T})$, on the unit circle $\mathbb{T}$, with a non-Carleman shift $\alpha : \mathbb{T} \to \mathbb{T}$ which has a finite set of fixed points, and where I is the identity operator, $c \in C^{n\times n}(\mathbb{T})$ is a continuous matrix function, $(U\varphi)(t) = \sqrt{|\alpha'(t)|}\varphi(\alpha(t))$ is the isometric shift operator and $P_\pm = \frac{1}{2}(I \pm S)$ are the complementary projection operators, with $(S\varphi)(t) = (\pi i)^{-1} \int_{\mathbb{T}} \varphi(\tau)(\tau - t)^{-1} d\tau$ the operator of singular integration with Cauchy kernel. It is supposed that all the eigenvalues of the matrix $c(t)$ at the fixed points of the shift, simultaneously belong either to the interior of the unit circle $\mathbb{T}$ or to its exterior. Under these and related conditions, estimates for the dimension of the kernel of the operator T, defined on the unit circle or on the one point compactification of the real line, are obtained. We obtain analogous estimates, under similar conditions, for an operator with polynomial coefficient relative to the shift operator. Particular cases and examples will be considered, which show that the proposed estimates are sharp.

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Statistics and Biometry
SUNDAY 5, 10.00–13.30. Poster 5.

Some contributions to the theory of discrete time birth-death branching processes

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Abstract

Branching process theory provides mathematical models for description of populations where an individual exists for a time and then may be replaced by others of a similar or different type. It is an active research area with theoretical interest and practical applications in several fields, specially in population dynamics.

Recently, in order to describe the demographic dynamics of socially-structured groups, Caron-Lormier et al. (2006) have introduced a new mathematical model based on a discrete time birth-death branching process. For each generation i, they consider that the random variable Z_i denoting the number of individuals born in that generation can be expressed as sum of the random variables U_i, V_i and W_i representing, respectively, the number of dead or living individuals having at least one ancestor alive, the number of living individuals whose ancestors are all dead, and the number of dead individuals whose ancestors are all dead.

In this work, such a birth-death branching scheme is extended considering deterministic control in each generation or assuming offspring probability distribution in varying environment. Also, the possibility of population-size dependent death process is studied. We derive several probabilistic results and we provide their biological significance. Finally, as illustration, we show some simulated examples.

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Statistics and Biometry
SUNDAY 5, 10.00–13.30. Poster 6.

Accessing the impact of ecological bias in ecological studies

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Abstract

Aggregated ecological regression studies rely on data given by measurements on groups of individuals rather than on the individuals themselves, potentiating the problem of ecological bias. The regression model for the more disaggregated level does not, in general, coincide with the regression model that we get for the grouped data. Information is lost with aggregation.

This problem can be caused or worsened by several reasons. Confounders for one level might not be so for the other and vice-versa; Problems of misclassification of the subjects into the different covariates levels, which is already a problem at individual level, might evolve to strange results when these are aggregated; The use standardized response data and non standardized covariates data; Different resolution scales for response and explanatory variables.

The consequences of the bias effects are essentially concerned with the interpretation of the estimated relationships from the aggregated to the individual level. In the most extreme case these models might only be used for describing purposes. This has lead to the idea that ecological regression studies should be used for *qualitative* assessments of associations between health data and covariates only, rather than *quantitative* associations.

The question we raise here is if there is an easy way to access ecological bias effects. Is ecological bias always a problem or can we sometimes neglect its effects? Can we get reliable indicators or even a test for answering that? The test must be constructed upon some measure of loss from the individual to the aggregated level, and should be set in a way that it can be carried out even if there is roughly any information on the individual level (or by a non-informative prior for it or by using reliable surrogate information).

We try to give an answer to these, in a setting of a hierarchical Bayesian framework, making use of some already existing solutions, proposed by other authors, to deal with this bias problem.

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Functional Analysis
SUNDAY 5, 10.00–13.30. Poster 7.

The *Dzhuraev*'s formulas and Bergman kind projections with continuous coefficients on domains with non smooth boundary

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Abstract

The poly Bergman projections do not admit usual representations in terms of singular integral operators and we will exemplify with different kind of domains in the mentioned conditions. Consequently a Fredholm symbol calculus for operators in C*-algebras of Bergman kind projections and continuous coefficients need to be constructed apart the theory of two-dimensional singular integral operators. We will present a symbol Fredholm calculus for the C*-algebra generated by a finite number of poly and anti-poly Bergman projections and multiplications operators by continuous functions on bounded domains without constrains on the boundary.

References

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Statistics and Biometry
SUNDAY 5, 10.00–13.30. Poster 8.

Meta-analytical assessment of predictive values of positive an negative immunohistochemical markers of desmoplastic melanoma

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Abstract

Immunohistochemistry plays an important role on distinguishing spindle cells and desmoplastic melanoma (DMM) from other tumors (neural, epithelial, hematologic, mesenchynal) they mimic (Olshier et al., 2008). Only four papers — including Soares de Almeida *et al.* (2008) — have findings based on more than 100 cases. DMM being rare, most of the published papers on the subject rely on scarce data (in the limit, in only 2 cases), carrying almost negligible isolated evidence, that can however be pooled up using a meta-analytical approach.

Evidence on the sensibility and specificity of the most usual markers (such as S-100, HMB45, MART–1/ Melan–A, tyrosinase and MIFT) is controversial (Xu et al, 2003). S-100 is, surely, the most sensitive, but its specificity is low. All the others seem to have higher specificity and lower sensibility, although as "negative marker" Melan-A may play an interesting role. Medic et al. (2006) discuss in depth the important issue of markers of circulating melanoma cells.

On the basis of the synthesis of the evidence provided namely in Chorny and Barr (2002), Granter et al. (2001), Iwamoto et al. (2001), Kucher et al. (2005), Orchard (2000), Radfar et al. (2006), Rib and McNutt (2003) and Sundram (2003), and in references therein, we discuss the predictive value of several markers, either isolated or in conjonction, in the diagnosis and prognosis of DMM.

Keywords and phrases. Systematic review, meta-analysis, sensibility, specificity, predictive value,

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Functional Analysis
SUNDAY 5, 10.00-13.30. Poster 9.

Maximum-likelihood estimation of functional parameters

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Abstract

Functional Statistics (see, for example, Ramsay and Silverman [2]) has become a useful context for the analysis of a wide range of complex biological systems. The high dimensionality of the data generated from these studies requires the development of new tools in Functional Statistics for efficient processing and analysis of such data sets. Orthogonal and biorthogonal transformations are usually applied to reduce dimensionality. The prediction problem is also formulated in a functional context when unobserved segments must be estimated. Funtional time series theory provides a suitable framework for solving this problem (see, for example, Bosq [1]).

Here, Autoregressive Hilbertian (ARH) models are considered to represent sequences of functional data (e.g. segments in biological sequences, spatio-temporal analysis of two-dimensional deformation and motion of cells from time series of digitized video images, etc.). Diagonalizations of these models in terms of a functional version of the Principal Oscillation Pattern (POP) decomposition have been obtained in Ruiz-Medina, Salmerón and Angulo [3], in the ARH(1) case, and in Salmerón and Ruiz-Medina [4] in the ARH(p), $p > 1$, case. In the mentioned papers, the implementation of the Kalman filtering algorithm is achieved in terms of the series defined by the POP coefficients of the process of interest, as well as in terms of the POP transformation of the conditional second-order moments involved. Both papers consider that the functional parameters in the formulation of ARH models are known.

However, in many applications, resarches also face the problem of parameter estimation of partially observed biological systems. Here, we address this problem, under the Gaussian assumption, for the ARH(p) model family. An EM algorithm is computed for estimation of the operators defining the functional parameters involved in the formulation of these models. Specifically, the expectation step (E-step) is computed from a forward Kalman filtering

recursion followed by a backward Kalman smoothing, previously implemented in terms of the POP decomposition derived in Ruiz-Medina, Salmerón and Angulo [3] and Salmerón and Ruiz-Medina [4]. The maximization step is then performed by applying standard techniques for multivariate Gaussian data, using the complete-data statistics computed in the E-step.

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Algebra and Algebraic Methods
SUNDAY 5, 10.00-13.30. Poster 10.

Generalized circulant matrix in coupled map lattice

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Abstract

A Coupled Map Lattice C.M.L. [1] is a chain of coupled elements (called oscillators), each situated on a discrete point of a lattice, whose individual dynamics is ruled by a discrete map $f(x)$. Broadly speaking, C.M.L. can describe systems consisting of a group of elements coupled by some kind of process, and at the same time, every element of the group is ruled by its own local dynamics. They are used in an extraordinary variety of applications in physics, biology, chemistry, social sciences and engineering modeling [3, 2].

The C.M.L. dynamics is given by

$$X_i(n+1) = (1-\alpha)f(X_i(n)) + \frac{\alpha}{m}\sum_{i=1}^{m} f(X_i(n)) \quad i = 1 \cdots m$$

and shows an ample variety of behaviors: travelling waves, synchronyzation, period doubling cascades, chaos, ets. All these behaviours have been observed numerically, although no analytical proofs of them have been given. We have proven [4] that for small enough values of the coupling term α the system can be linearized and analitycal results can be obtained from this linearization. The main result is that the solution of the system is ruled by the

fixed points of the individual behaviour of each oscillator plus perturbation (given by A_j) obtained from the system:

$$\begin{pmatrix} -f'(x_1^*) & 1 & 0 & 0 & \cdots & 0 \\ 0 & -f'(x_2^*) & 1 & 0 & \cdots & 0 \\ 0 & 0 & -f'(x_3^*) & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & \cdots & -f'(x_p^*) \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_p \end{pmatrix} = \alpha \begin{pmatrix} -x_2^* + \frac{1}{p}\Sigma_{j=1}^{p} x_j^* \\ -x_3^* + \frac{1}{p}\Sigma_{j=1}^{p} x_j^* \\ -x_4^* + \frac{1}{p}\Sigma_{j=1}^{p} x_j^* \\ \vdots \\ -x_1^* + \frac{1}{p}\Sigma_{j=1}^{p} x_j^* \end{pmatrix}$$

(where x_j^* is a fixed point of f^p, and f^p denotes $f \underbrace{\circ \cdots \circ}_{p} f$).

The matrix of this linear system is a generalization of circulant matrix, whose inverse is given by:

$$\begin{pmatrix} f'(x_2^*)\cdots f'(x_p^*) & f'(x_3^*)\cdots f'(x_p^*) & f'(x_4^*)\cdots f'(x_p^*) & \cdots & 1 \\ 1 & f'(x_3^*)\cdots f'(x_p^*)f'(x_1^*) & f'(x_4^*)\cdots f'(x_p^*)f'(x_1^*) & \cdots & f'(x_1^*) \\ f'(x_2) & 1 & f'(x_4^*)\cdots f'(x_p^*)f'(x_1^*)f'(x_2^*) & \cdots & f'(x_1^*)f'(x_2^*) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f'(x_2^*)\cdots f'(x_{p-1}^*) & f'(x_3^*)\cdots f'(x_{p-1}^*) & f'(x_4^*)\cdots f'(x_{p-1}^*) & \cdots & f'(x_1^*)f'(x_2^*)\cdots f'(x_{p-1}^*) \end{pmatrix}$$

The different behaviours observed numerically in C.M.L. are ruled by this inverse matrix.

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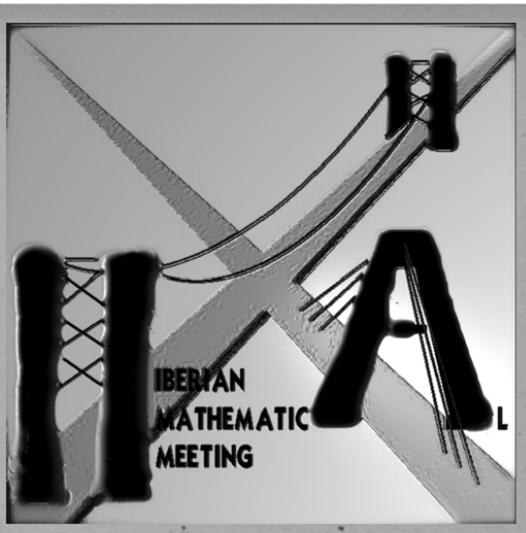

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