D-modules in positive characteristic and Frobenius descent

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Abstract

Let $R = k[x_1, \ldots, x_d]$ be the ring of polynomials in a finite number of variables over a field k and let $D_{R|k}$ be the corresponding ring of k-linear differential operators. The theory of $D_{R|k}$-modules has been successfully applied in Commutative Algebra in order to study local cohomology modules due to the fact that, despite not being finitely generated as R-modules, they are so when considered as modules over $D_{R|k}$.

When k is a field of characteristic zero, $D_{R|k}$ is the ring extension of R generated by the partial derivatives $\{\partial_i := \frac{d}{dx_i} \mid i = 1, \ldots, d\}$. In this setting, G. Lyubeznik [3] proved some finiteness properties of local cohomology modules using the fact that they are holonomic. This is a nice class of $D_{R|k}$-modules satisfying some good properties, in particular they have finite length.

When k is a field of characteristic $p > 0$, $D_{R|k}$ is the ring extension of R generated by the set of differential operators $\{\partial_i^{<t>} := \frac{1}{t!}\frac{d^t}{dx_i^t} \mid t \in \mathbb{N} \ , \ i = 1, \ldots, d\}$ so it is no longer a Noetherian ring. Therefore, the theory of $D_{R|k}$-modules in positive characteristic do not behave as in the case of characteristic zero as it was pointed out in [1].

The aim of this talk is to give a better understanding of $D_{R|k}$-modules in positive characteristic. In particular, we are interested in the notion of holonomic modules and its comparison with the category of F-finite F-modules introduced by G. Lyubeznik [4]. The main ingredients we are going to use are the rings of differential operators of level e given by P. Berthelot [2] and the so-called Frobenius descent.

References

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