Parameter dependence of solutions of differential equations on spaces of distributions

José Bonet
Universidad Politécnica de Valencia
e-mail: jbonet@mat.upv.es

Pawel Domanski
University Poznań (Poland)
e-mail: domanski@amu.edu.pl

Abstract

We report on joint work with Pawel Domański. The lecture is based on our papers [1, 2] and on the paper by Domański [3].

We investigate whether for every linear partial differential operator with constant co-efficients $P(D) : \mathcal{D}'(\Omega) \to \mathcal{D}'(\Omega)$, $\Omega \subset \mathbb{R}^d$ convex open, and every family of distributions $(f_\lambda)_{\lambda \in U} \subseteq \mathcal{D}'(\Omega)$ depending holomorphically (or smoothly C^∞ or in a real analytic way) on the parameter λ running through an arbitrary Stein manifold U (or C^∞-manifold U or real analytic manifold U), there is an analogous family $(u_\lambda)_{\lambda \in U}$ with the same type of dependence on $\lambda \in U$ such that

$$P(D)u_\lambda = f_\lambda \qquad \forall \lambda \in U. \tag{1}$$

In fact we provide a very efficient and general method for solving (1) for various types of dependence on λ. Our results are based in a deep analysis of the splitting of short exact sequences of PLS-spaces. This is a large class containing all the spaces which appear in the analytic applications of linear functional analysis, like spaces of (ultra)distributions, real analytic functions and holomorphic functions.

References

[1] J. Bonet, P. Domański, *Parameter dependence of solutions of differential equations on spaces of distributions and the splitting of short exact sequences.* J. Funct. Anal., **230** (2006), 329–381.

[2] J. Bonet, P. Domański, *The splitting of short exact sequences of PLS-spaces and smooth dependence of solutions of linear partial differential equations.* Advances in Math., **217** (2008), 561-585.

[3] P. Domański, *Real analytic parameter dependence of solutions of differential equations.* Preprint, 2008.