Growth and Extinction of Populations and Individuals in Randomly Varying Environments

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Abstract

In a randomly varying environment, the per capita growth rate (abbreviately growth rate) of a population can been described by an "average" rate $g(N)$ (usually dependent on population size N) perturbed by a white noise (as a reasonable approximation to a noise with low correlations). So, with $N = N(t)$ being the population size at time t, we consider the general model

$$\frac{1}{N}\frac{dN}{dt} = g(N) + \sigma\varepsilon(t)$$

where $\sigma > 0$ is the noise intensity and $\varepsilon(t)$ is a standard white noise.

Denoting by $W(t)$ the standard Wiener process, we can write the model in the standard form of a stochastic differential equation (SDE)

$$dN(t) = g(N(t))dt + \sigma N(t)dW(t).$$

These models have been studied in the literature for specific functional forms of the "average" growth rate" g (like, for example, the logistic model $g(N) = r(1N/K)$). Since it is hard to determine the "true" functional form of g, one wonders whether the qualitative results (concerning population extinction or existence of a stationary density) are model robust. We have managed to prove the usual qualitative results for a general function g satisfying only some basic assumptions dictated by biological considerations and some mild technical assumptions (see [1], [2]). From the applied point of view, it was embarrassing that the two main stochastic calculus, Itô and Stratonovich, lead to apparently different qualitative results regarding important issues like population extinction and that led to a controversy in the literature on which calculus is more appropriate to model population growth. We have resolved the controversy (see [3], [4]) by showing that g means different types of "average" growth rate according to the calculus used and the apparent difference was due to the wrong implicit assumption that g represented the same "average". Taking

into account the different meaning of g, there is no difference (qualitative or quantitative) between the two calculi.

The results were then generalized to the case of density-dependent noise intensities $\sigma(N)$ satisfying mild assumptions (see [5], [6]).

We have also recently considered (with Patrcia A. Filipe, see [7]) SDE models for the individual growth from birth to maturity of the size (weight, volume, length, ...) of individual animals (or plants) and we briefly report some results with applications to cattle breeding.

References

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