Approximation and entropy numbers in Besov spaces of generalized smoothness

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Abstract

Let X and Y be Banach (or quasi-Banach) spaces and let $T \in \mathcal{L}(X, Y)$ be a compact operator. In order to measure "the degree of compactness of T" one can use the asymptotic decay of the sequence of approximation numbers of T or the decay of the sequence of entropy numbers of T. In general, the behaviour of these sequences is quite different. However, as we shall show, under certain assumptions the two sequences behave in the same way. Then we shall use this result to determine the exact asymptotic behaviour of entropy and approximation numbers of the limiting restriction operator $J : B^{s,\psi}_{p,q_1}(\mathbb{R}^d) \to B^{s}_{p,q_2}(\Omega)$, defined by $J(f) = f|_\Omega$. Here Ω is a non-empty bounded domain in $\mathbb{R}^d, \psi$ is an increasing slowly varying function, $0 < p < \infty, 0 < q_1, q_2 < \infty,\ s \in \mathbb{R}$, and $B^{s,\psi}_{p,q_1}(\mathbb{R}^d)$ is the Besov space of generalized smoothness given by the function $t^s\psi(t)$.

The talk is based on a joint paper with Thomas Kühn which is going to appear in the Journal of Approximation Theory