

On finite complete rewriting systems and completely 0-simple semigroups

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Abstract

Given a binary relation R on a free semigroup over an alphabet A we can associate to it a (string-)rewriting system by considering the single-step reduction \rightarrow_R . At the same time we can associate to R the Thue congruence $\stackrel{*}{\leftrightarrow}_R$ generated by R , and consequently the semigroup resultant of the quotient of A^+ by $\stackrel{*}{\leftrightarrow}_R$. Regard R as a rewriting system and study the properties of the single-step reduction corresponds to a Theoretical Computer Science point of view, while the study of the semigroup quotient $A^+ / \stackrel{*}{\leftrightarrow}_R$ corresponds to an Algebraic point of view.

The study of the properties of the single-step reduction, namely finiteness and completeness (that is, noetherian and confluent), can be used to solve word problems among other algebraic decision problems. On the other hand completely 0-simple semigroups are one the most important theoretical semigroup constructions, since it is known by the structure theorem of Rees that every completely 0-simple semigroup is isomorphic to some $I \times \Lambda$ Rees matrix semigroup $\mathcal{M}^0[G; I, \Lambda; P]$ over a group G .

In this talk we intent to briefly discuss the above notions and present results relating the mentioned properties of a rewriting system associated to the group G with the same properties on the completely 0-simple semigroup $\mathcal{M}^0[G; I, \Lambda; P]$.