

# Ensuring the additivity property for estimating the total tree biomass

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## Abstract

The estimation of tree biomass is a very important issue for both practical forestry and scientific purposes. As forestry activities mainly affect the exchange of carbon dioxide between the land and the atmosphere, the management of these activities offers an opportunity to isolate atmospheric carbon that constitutes a portion of a regions greenhouse gas contribution. Then, estimation of aboveground tree biomass is an essential aspect of carbon stocks studies and the effects of deforestation and carbon sequestration in the global carbon balance. The most common procedure for estimating tree biomass is the use of direct measurements of biomass and regression techniques. In forestry research, the development of allometric equations or models relating the weight of the different parts of the tree with other quantities, usually the stem diameter, is essential to predict the tree biomass. However a major drawback arises because the predictions of the weight, obtained for the different tree components, do not add up to the predictions derived from modelling the total tree weight, that is, the property of biomass additivity is not fulfilled. This means that the regression functions for the different tree components are not consistent with each other. The main goal of this work is to propose and compare alternative techniques to ensure additivity of the regression predictions providing some advantages over the widely used logarithmic model. Firstly, non linear seemingly unrelated equations are derived, because this model accounts for statistical dependencies among the data. Secondly, we consider the application of segmented regression because an apparent non linear relationship between two variables can be split up into two or more linear regressions. The procedure is very appealing because it can offer a simpler and clearer interpretation than more complex models. However, if the break point of the segmented regression is not the same for all the tree components, the

additivity property is not fulfilled. Then, we propose the calculation of a common break point for all the tree components such that the desired additivity property is attained. Finally, as the distribution of the weights is asymmetric, we also propose to fit quantile segmented regression in order to provide a more complete picture of the whole distribution. The procedures are illustrated with a data set of 42 beech trees (*Fagus Sylvatica*) from a forest in the northern part of Navarra, Spain.