The Cuntz semigroup and its impact into classification

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Abstract

In 1976, Elliott, building on work of Glimm and Bratteli, gave a classification of countable limits of semisimple algebras over algebraically closed fields. This in fact applies to classify, in terms of ordered K-Theory, all approximately finite dimensional C*-algebras ([3]). His discovery led him to conjecture, around 1989, that a wide class of simple C*-algebras would be classified by K-theoretic invariants, and bolstered his claim by showing with D. Evans that the algebras known as irrational rotation algebras were so classified ([4]).

The so-called Elliott Programme has enjoyed a resurgence of late, owing to the discovery by M. Rørdam ([7]) and later by A. S. Toms ([8]) that K-Theory does not suffice for the classification of all separable, simple, amenable C*-algebras. There are two ways forward: restrict the class of algebras considered or enlarge the proposed invariant. Both of these courses have been pursued vigorously over the past three or four years, leading to several breakthroughs in Elliott's Programme.

The following question has received a lot of attention recently: "Can one characterize the largest class of simple, separable, amenable C*-algebras for which Elliott's original conjecture holds? " Two properties related to this stand out: $\mathcal{Z}$-stability and strict comparison of positive elements.

Here, $\mathcal{Z}$ is the so-called *Jiang-Su algebra* ([5]). The interest of the first property stems from the fact that taking a tensor product with $\mathcal{Z}$ is inert at the level of K-Theory. The second property of interest is related to an object called the Cuntz semigroup. This can be associated to any C*-algebra and its recent popularity is due to its extreme sensitivity as an invariant. This is due to the fact that it is able to distinguish simple, separable, amenable C*-algebras with the same K-Theory and tracial space. The property of strict comparison says, roughly, that the natural partial order on the Cuntz semigroup is determined by certain states. This property often – conjecturally, always – implies that the Cuntz semigroup can be recovered functorially from K-Theory and traces (see [1], [2], [6]).

The purpose of the talk will be to give an account of these recent goings-on.

References

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