On some inverse problems in the theory of orthogonal polynomials and their connections with Jacobi operators

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Abstract

Given a positive Borel measure whose support is an infinite set of points in the real line the problem of the construction of an orthogonal polynomial sequence (OPS) with respect to that measure is known as *direct problem*. An *inverse problem* (IP) consists in finding the orthogonality measure when only some information is known about a given OPS (e.g., via a three-term recurrence relation, or via the knowledge of some algebraic or analytical relations between the given OPS and some other ones). In this talk we will consider several models of inverse problems in the Theory of Orthogonal Polynomials. For instance:

(IP) Let $(p_n)_n$ be a given monic OPS and k a fixed positive integer number such that $k \geq 2$. We want to analyze conditions under which this OPS is constructed from a polynomial mapping in the following sense: to find another monic OPS $(q_n)_n$ and two polynomials π_k and θ_m, with degrees k and m (resp.), with $0 \leq m \leq k-1$, such that

$$p_{nk+m}(x) = \theta_m(x) q_n(\pi_k(x))$$

for all $n = 0, 1, 2, \cdots$.

Assuming that the support of the positive Borel measure with respect to which $(p_n)_n \geq 0$ is orthogonal is a compact set, we give explicitly this measure in terms of the orthogonality measure for the OPS $(q_n)_n$. One gets orthogonality on several intervals of the real line, with possible mass points located at the gaps between these intervals. A connection with the so-called sieved OPS on the unit circle is made, leading to families of OPS on the unit circle with an orthogonality measure supported on several arcs of the unit circle. Our results enable us to recover several known results in the literature, including some ones which arise from connections between this kind of transformation laws and the spectral theory of self-adjoint Jacobi operators.