

On a family of circular distributions due to Batschelet

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Abstract

Circular data arise in scientific disciplines as diverse as meteorology, the earth sciences, biology, medicine and the political sciences. For instance, they might represent the directions of prevailing winds in the vicinity of a wind farm, the orientations of fault lines in geological bedrock, the directions of migrating birds, the degree of flexibility of the legs of injured cyclists, or the times of violent attacks in occupied Iraq.

In recent years, various families of symmetric unimodal distributions have been proposed in the literature as models for circular data. Three particularly flexible ones are the wrapped symmetric stable ([2], Section 2.2.8), wrapped t ([4]) and the Jones–Pewsey ([3]) families. Each of these families has three parameters controlling the distribution’s mean direction, concentration/scale, and shape.

Here, consideration is given to an alternative three-parameter family of symmetric unimodal circular distributions originally proposed by Batschelet ([1], p. 288) as an extension to the von Mises distribution. Batschelet’s family is an interesting one as it not only contains sharply peaked distributions, like the three families already mentioned, but also distributions which are far more flat-topped than the von Mises. The family’s fundamental properties are derived, and an in-depth comparison made between it and its direct competitors. Likelihood based techniques are developed which can be used to conduct point and set estimation for the parameters as well as hypothesis testing. Finally, an analysis of a data set taken from the geological literature is presented which illustrates how the family and its direct competitors can be employed in the search for parsimonious models for circular data.

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Key words and phrases: Flat-topped distributions; likelihood based inference; sharply peaked distributions; symmetrical unimodal distributions; von Mises distribution; wrapped symmetric stable family.

References

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