

# Competition models and other probabilistic cellular automata

Gerardo Sanz  
Departamento de Métodos Estadísticos  
Universidad de Zaragoza  
e-mail: gerardo@unizar.es

## Abstract

Consider  $M$  distinct species living in the space  $\mathbf{Z}$  ( $\mathbf{Z}^d$ ). Each site of the space can be occupied by only one individual. At each time step, each individual produces one offspring that goes to colonize a neighboring site, killing the old particle that lived in that site and competing against the offspring of other individuals. Is there an equilibrium measure? Is there, in long-term, coexistence of different species?

This competition model and other biological or physical models can be seen as Probabilistic Cellular Automata (PCA).

Probabilistic Cellular Automata (PCA) are discrete time stochastic processes with state space  $\mathbf{X} := W^{\mathbf{Z}^d}$ , where  $W = \{1, \dots, M\}$  and  $d \geq 1$ , with finite range interactions on the integer lattice  $\mathbf{Z}^d$ , that is, at each point  $z \in \mathbf{Z}^d$  there is a particle (individual) which takes values in the set  $W$  and that changes its value according some transition rule which depends on the values of its neighbor particles.

Both in continuous and discrete time processes, duality is a powerful tool for studying these systems ([1]), since it provides relevant information about the evolution of the process under consideration from the study of other simpler process, the dual process.

In this work, we introduce a new form of duality for multistate PCA. This new idea of duality includes the classical duality and allows us to study processes that do not have a dual in the classical sense.

Using this duality, ergodicity results for wide classes of PCA are obtained. The results provide the existence of an equilibrium measure for the competition model described above and for many other models as multi-opinion models or Domany-Kinzel models, solving some open problems in the literature ([2]).

This is a joint work with F.J. López and M. Sobottka.

## References

- [1] López, F. J. and Sanz, G. , *Duality for general interacting particle systems*. Markov Processes and Related Fields, **6** (2000), 305-328.

- [2] López, F.J.; Sanz, G. and Sobottka, M. , *Dualities for multi-state probabilistic cellular automata*. Journal of Statistical Mechanics, **P05006** (2008), 1-22.